OPTIMIZING THE PRE-HOSPITAL MOBILE CARE SYSTEM: A CASE STUDY

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▪ ABSTRACT: Freidenfields (1980) introduced for the modeling of several transport systems demands the concepts of queueing theories and studied the problem of the capacity expansion of the transport system as a random process of life and death, showing that it is possible to adapt the stochastic model of demand growth into a deterministic model. Souza (1996) applied this theory to predict the expansion of the emergency care systems. The modeling of the supply of U_s integrated into the hospitals – emergency care and inter-hospital removals – despite being considered a restricted market service, as new solutions are developed new knowledge is aggregated into an increasingly lower cost (GOLDBERG, 2004). The dimensioning, allocation and distribution of the supply of U_s developed for the pre-hospital mobile care system, utilizing data based on the Brazilian situation, is a field that deserves extreme attention. That will allow the assessment of the present situation and can lead to new routes in terms of public policies. Thus, the distribution of service stations of the regulation centers represents the ordering and orienting element of the State Systems of Urgency and Emergency. These centers must be structured in all levels, organizing the relation between several services, qualifying the flux of patients in the system and generating an integrative gateway for the hospitals, by which distress signals are received, evaluated and ranked. These rules must be followed by all services, both public or private. It can be mentioned, as an example, that for the emergency services a widely used measure is the maximization use of the U_s or the minimization of response time (TR), between any user of the transport system and the nearest hospital.

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1 Introduction

The first models developed for care emergency services were deterministic according to (CRONK *et al.*, 1986; GOODMAN *et al.*, 1986) and were important for planning and investigation analysis, ignoring the stochastic considerations about the problem. A revision of the specific models of localization of service unit (U_s) for medical emergency is addressed by (BROTCORNE *et al.*, 2003).

A great disadvantage of these models is that they come from the hypothesis that U_s are available when requested, which is not always reasonable in practical applications. The congestion in emergency care services, that can induce the unavailability of U_s, motivated the development of probabilistic models. In probabilistic modeling of emergency services, some simplifying hypotheses allow the use of mathematical programming. Nonetheless, in more general situations some hypotheses are not applicable; conducting the treatment of these problems through the use of stochastic process.

Various probabilistic models have been developed, considering the stochastic nature of the events (accidents), such as the fact that U_s operate as servers in a queue system and sometimes are not available for operation, as occurred in the case study developed by GOLDBERG *et al.* (1990) and CHING (1997). The hypercube model, proposed by Larson (1982), and analyzed by many authors (SWERSEY, 1994) is an important mathematical tool for the emergency systems planning, especially the urban systems in which the U_s moves to assist in some type of accident. The model is adequate for analyzing coordinated or centralized systems, where the user who needs assistance is demanded through a care center system. The system manager then dispatches the U_s of a facility that is close to the occurrence. In the case of no U_s being available, the request is put into a waiting queue so that it can be fulfilled as soon as the U_s is available.

The applications of the hypercube model are innumerable. In Brazil, some examples are: the ambulance localization in a tract of the BR-111 – SC road (GONSALVES *et al.*, 1994, 1995), the balancing of the usage factor of ambulances in the "Asphalt Angels" system of the Presidente Dutra road (MENDONÇA and MORABITO, 2000, 2001), and the performance analysis of the SAMU-192 system of Campinas – São Paulo (TAKEDA *et al.*, 2004).

Initially the intention is of calculating the minimum number of U_s necessary from each region R, from estimated parameters of this region, in a way that there is enough confidence to not have any missing U_s for assisting the patient of a specific region.

To maximize the efficiency of the assistance, it is made necessary to adequately spread the U_s of a city, minimizing the response time TR, defined as the time between the request of U_s and their arrival at the accident location. The most severe problem is that, in general, between the reception of the request in the screening center or service station and the sending of the service unit – U_s, some time is unnecessarily lost as a result of a lack of a perfect operational system in the screening center.

2 Dimensioning of stations/Integration centers – Hospital beds

Consider a given region R with initial population h. Each user who belongs to R has constant long time coordinates $(\lambda,\ \mu)$, defined by:
λ: user accident average rate;
μ: off-care user average rate.

(λ) and (μ) are denominated average rates because it is necessary to estimate them from a known population. The accident event is defined as any entry in the hospital through a service unit – U_s. The following hypotheses are premises for the system demand dimensioning:

1. User of region R can only suffer accidents by being out of hospitalization;
2. All injured and not injured users (operational) are considered as random independent variables;
3. Individuals who arrive at hospital using a U_s as means of transport are considered users of the system;
4. Population h is considered a function that does not vary during the time of the modeling.

By the first hypothesis we do not compute as accidents the removals (user patients that are transferred from a hospital to another), because they are interactions that do not alter the number of hospitalized individuals (system users). The definitions hereafter are to be considered: $TMH = \lambda^{-1}$, average time of system user's hospitalization and $TMF = \mu^{-1}$, average duration time of a not injured patient – user's out of system (healthy user).

Considering TV as the travel time of a U_s, it can be then written that $\lambda^{-1} = TV + TMH$, because at the moment the individual suffers an accident, he/she becomes the system user and at the moment he/she gets out of the hospital, the user is cured, exiting the system. An approximation of this equation to reality can be made, because as quoted in (CORDEIRO, 2012) the TMH in Brazil varies from $6,10$ days to $8,65$ days; in other words, it is possible to consider that $TMH >> TV$. Thus, this variable can be initially overlooked by doing, $TMH = \lambda^{-1}$. The presumption that $TV = 0$ is coherent with the third hypothesis, due to the fact that possible improvements in the $U_s s$ services of type GPS are not being considered because these only alter the TV, that is being relaxed in the initial modeling.

Consider now X_t defined as a random variable that represents the number of cured citizens or non-injured in the region R in the period of time t. The maximum user population of the system (maximum demand of the system – h_{max}) represents the whole community (population h) of region R, while the user population of the system (effective demand) refers to the injured individuals (users) of the region R that were transferred to a hospital using U_s as means of transport. Being j the state it can be said that the system is in state j – E_j in time t if, and only if, $X_t = j$. In this way, it can be defined that the probability of an individual in state j is cured is represented by: $P_j(t) = P(X_t = j)$.

First and second hypotheses conditioned to the system user rates – injured individuals (λ_j) and operational – cured individuals (μ_j), represent a process of life and death, respectively. A simplified equation, for the calculation $P_j(t) = P(X_t = j)$, is detailed in (CORDEIRO, 2002), in other words:

$$P_j = C_h^j \frac{\left(\frac{\lambda}{\mu}\right)^j}{\sum_{j=0}^{h} C_h^j \left(\frac{\lambda}{\mu}\right)^j}. \tag{1}$$

Defining p as being the probability of an individual in R of being cured in the long run, in other words, p is the proportion of the time he remains out of the system. This equation represents the quotient between the TMF and $(TMF + TMH)$. Therefore,

$$p = \frac{\lambda}{\lambda + \mu}. \tag{2}$$

Taking the complement of the Equation (2), it turns out that $(1 - p) = \left(\frac{\mu}{\lambda + \mu}\right)$, which represents the probability of an individual in region R of becoming a user of the system. It is important to highlight that these probabilities are stabilized in the long run. This way, after the replacement of the term $\left(\frac{\lambda}{\mu}\right)$ in Equation (1), we come from the probability function of P_j, to a very known distribution, which is the binomial distribution with success probability p and failure probability $(1 - p)$.

$$P(X = j) = C_h^j \; p^j \; (1 - p)^{h-j}, \quad j = 0, 1, \ldots, h. \tag{3}$$

After the definition of the X distribution, it is important to calculate the statistics distribution. This way, the number of cured individuals (X) after the system reaches the stable equilibrium follows a binomial distribution of average $E(X) = hp$ and variance $V(X) = hp\;(1 - p)$.

Being Y another random variable, which represents the number of users relocated from the system to a hospital, using U_s as means of transport. Then, the population variable can be written as $H = X + Y$; and as Y is a binomial, H also follows the binomial law (HOEL, 1996). Therefore, the probability of j injured individuals in region R being users of the system can be calculated by,

$$P(Y = j) = C_h^j (1-p)^j p^{h-j}, \quad j = 0, 1, \ldots, h. \tag{4}$$

Consider now $Y = \sum_{i=0}^{h} Y_i$ and Y_i as independent. The "dummy" variable concept is adopted, in which $Y_i = 1$ if the ith individual is a user of the system (relocated to the hospital, using U_s as means of transport) or $Y_i = 0$, if not. In virtue of h being big in comparison with Y, we understand by the central limit theorem (HOEL, 1996) that the random variable Y will be a normal one with the following representation: $Y \sim N(h(1-p),\ h(1-p)p)$.

In order to gain a more accurate picture, suppose that the population of region R is $h = 15.000$ – order of magnitude 10^4; it can be understood that the probability of an individual to be cured (not injured) in region R is of $p = 0,999$. So it is important to know what is the probability of an individual never being a user of the system – there never is an accident with this individual inside the region R, in a way that, by suffering an accident, he/she will be relocated to a hospital using U_s as a means of transport. Therefore we have,

$$P(Y \leq 0) = P\left(Z \leq \frac{0 - h(1-p)}{\sqrt{h(1-p)p}}\right), \tag{5}$$

where $Z \sim N(0,1)$. Making the appropriate substitutions, we can find that $P(Y \leq 0) = P(Z \leq -3,8749) = 0,0001 \neq 0$. Thus, using a $Z \sim N(0,1)$ – reduced normal , does not bring significant results, because it imputes $P(Y \leq 0)$, a $0,0001$ probability – only 10 times smaller than the order of magnitude of $(1-p) = 0,001$, to an event that surely can never happen.

Consequently, for bigger improvements in the pre-hospital care services (MCALEER and NAQVI, 1994) and (TAVAKOLI and LIGHTNER, 2003) suggest that the regions corresponding to a city division must be smaller in a way that: $h \downarrow$ implies a $P(Y \leq 0) \uparrow$. In order to correct the distortion of $P(Y \leq 0)$, it is necessary to adopt the following procedure: instead of Y being treated as a normal $Z \sim N(0,1)$, treating Y as a truncated normal, which probability density function (p.d.f.) is given by:

$$f(y) = c[2\pi\ V(Y)]^{-0,5} \exp\left\{\frac{-1}{2}\left[\frac{Y - h(1-p)}{\sqrt{V(Y)}}\right]^2\right\}, \tag{6}$$

with $Y > 0$ and c representing the correction constant for the p.d.f. of Y, calculated by: $c = \Phi^{-1}\left(\frac{E(Y)}{\sqrt{V(Y)}}\right)$.

To dimension h it is necessary to know the number of hospital beds (n_L) being supplied by the hospital beforehand, which are situated in the region R. Knowing n_L, the problem consists now in calculating the maximum user population of the transport by U_s, in a way that the probability of not having missing hospital beds is known (α).

The solution for this problem will be important in the division of a city in assistance zones (ZA) which will correspond to the hospitals in a one-to-one relation. That way, knowing n_L, we can arrive to the calculation of its user population and the corresponding zone. Consider that $Y \sim N(h(1-p), h(1-p)p)$. Then, we can calculate the probability $P(Y \leq n_L) = \alpha$, by means of:

$$P(Y \leq n_L) = c\,(2\pi\ h(1-p)p)^{-0,5} \int_0^{n_L} \exp \frac{-1}{2} \left(\frac{y - h(1-p)}{\sqrt{V(Y)}} \right)^2 dy. \qquad (7)$$

Resolving the Equation (7) we can obtain the estimation of the n_L equation.

$$n_L = h(1-p) + \sqrt{h(1-p)p}\ \Phi^{-1} \left[1 + (\alpha - 1)\ \Phi \left(\sqrt{h(1-p)p^{-1}} \right) \right]. \qquad (8)$$

The Equation (8) is very general, because it allows to calculate n_L from h and vice-versa under any circumstances. However, if the condition $h \geq 16 \left(\frac{p}{1-p} \right)$ is satisfied, the problem inverts, and the calculation is made by $Z \sim N(0,1)$. That way, knowing h and p, we can now calculate n_L by:

$$n_L = h(1-p) + \sqrt{h(1-p)p}\ \Phi^{-1}(\alpha). \qquad (9)$$

Considering that there must not be any beds missing in the hospitals, then $\alpha = 1$. Therefore, any of the equations can be used for the definition of n_L. And based on the normal distribution table, $\Phi^{-1}(\alpha) \geq 4$, that means, $n_L \geq 4\sqrt{h(1-p)p} + h(1-p)$. On this case, the search for the minimum value of n_L is obtained from the equality:

$$n_{L_{min}} = h(1-p) + 4\sqrt{h(1-p)p}. \qquad (10)$$

The city of Recife, currently, provides 8.089 hospital beds; in practice when dividing the city in ZAs (zones), The n_Ls being provided are certainly greater than 40. This value was calculated considering the extreme case, in which each Zone represents a city neighborhood (there are in Recife at least 200 neighborhoods). Setting the average variability of $(1-p)$ between $4,2 \times 10^{-4}$ (national average) and $1,0 \times 10^{-2}$, the condition $h \geq 16 \left(\frac{p}{1-p} \right)$ will always be verified. Thus, the search for h always comes from the Equation (8). Now, doing the proper isolation of h, we have:

$$h = \frac{1}{4} \frac{p}{(1-p)} \left\{ \left[\sqrt{(\Phi^{-1}(\alpha))^2 + \frac{4n_L}{p}} \right] - \Phi^{-1}(\alpha) \right\}^2. \qquad (11)$$

In case of interest – no missing beds, $\alpha = 1$. Replacing $\Phi^{-1}(1) \geq 4$ in Equation (11), the value of h is determined according to n_L, and vice-versa and, therefore $n_L \geq h(1-p) + 4\sqrt{h(1-p)p}$. The maximum user population value (maximum demand) h_{max} is calculated by considering the strict equality in the above equation.

$$h_{max} = \frac{\left(\sqrt{4p + n_L} - 2\sqrt{p}\ \right)^2}{(1-p)}. \qquad (12)$$

From Equation (11) it is concluded: if $\alpha \uparrow$ then $h \downarrow$; when $\alpha \to 0$, $h \to +\infty$. In Table 1, the results of the simulation for obtaining the values of h are found from $(1-p)$, α and n_L. The quantile function of the normal $\Phi^{-1}(\alpha) = z$, being $Z \sim N(0,1)$ was calculated using the routines "Cumulative probability" and "Inverse cumulative probability" (MINITAB, 2011). Values of $\Phi(z)$ were generated between 0 and 3.9999. Afterwards $\Phi^{-1}(\alpha)$ was selected for $\alpha = 0,8$, $\alpha = 0,95$ and $\alpha = 0,99999$ (h_{max} – maximum user population or maximum demand). The values of $(1-p)$ – the probabilities of an individual being a system user were fixated in $0,001$, $0,002$, $0,004$, $0,008$ and $0,01$.

Table 1 - The estimated values for h in function of $alpha$, n_L and $(1-p)$

α	Φ^{-1}	n_L	$h(0,001)$	$h(0,002)$	$h(0,004)$	$h(0,008)$	$h(0,1)$
0,80	0,84378	250	237.016	118.511	59.259	29.633	23.707
0,95	1,6482	250	225.274	112.643	56.327	28.170	22.538
0,99997	3,99997	250	194.275	97.150	48.587	24.306	19.450
0,80	0,84378	650	628.851	314.431	157.221	78.616	62.895
0,95	1,6482	650	609.335	304.677	152.349	76.184	60.951
0,99997	3,99997	650	555.750	277.897	138.970	69.507	55.614
0,80	0,84378	1000	973.684	486.848	243.431	121.722	97.380
0,95	1,6482	1000	949.245	474.635	237.330	118.677	94.947
0,99997	3,99997	1000	881.313	440.684	220.370	110.213	88.182

The World Health Organization (WHO) recommends for the hospital conjuncture that the number of hospital beds of a city can be determined by the ratio of 3 beds for each 1.000 inhabitants. In Brazil, the adopted ratio is usually 2,53 beds for each 1.000 inhabitants (IBGE, 2005). However, each country adopts a different ratio. It is a fact that the conditions for an inhabitant to suffer an accident and be taken to a hospital vary according to the region.

In major cities the probability is way higher than in the respective states. Under the Equation (12), if n_L is a lot bigger ($n_L \to +\infty$) and α is any positive value greater than zero, we have: $h = n_L\ (1-p)^{-1}$ or $(1-p) = n_L\ h^{-1}$ and, this way, $(1-p)$ can be defined as being the bed per inhabitant unit factor, and that way, the number of beds is equal to the expected number of injured individuals $E(Y)$ (system users).

Therefore, it can be concluded that the greater n_L is, the smaller the average percentage of idleness will be (tendency of system user population being equal to the bed supply); this average idleness time tends towards zero when $n_L \to +\infty$. That way, the usage of hospital beds percentage is given by $\dfrac{h\ (1-p)}{n_L}$. Thus, for α and $(1-p)$ being constants, the expected percentage of idle beds is $\left[1 - \dfrac{h\ (1-p)}{n_L}\right] 100$, which increases when n_L increases.

3 Estimates of $P - \hat{p}$

Consider Y_t be the random variable that represents total the number of injured individuals (system users) and hospitalized in the period of time t and being I_n the number of hospitalized users of a given region R's hospital in the period of time $\Delta t = (t_2 - t_1)$, which in regards to the survey made by IBGE is annual (365 days).

Consider, now, I_{n_1}, the number of hospitalized users remaining in the period of time t_1. The area of the curve Y_t between t_2 and t_1, according to Figure 1, represents the total of injured users and hospitalization in period of time Δt (user x day). So, the average time of hospitalization – TMH for all individuals (cured individuals + system users) of the region R, after the stable equilibrium, will be calculated by:

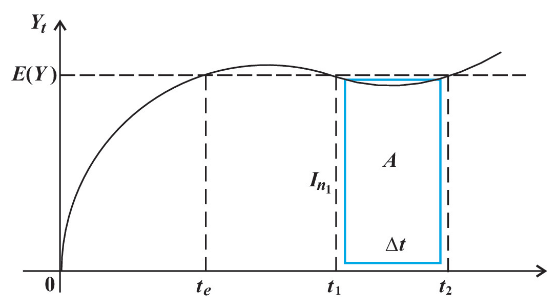

Figure 1 - Curve representation of (Y_t) x t.

$$TMH = \frac{(A - I_{n_1} TMH)}{(I_n - I_{n_1})}. \tag{13}$$

Solving it, we come to: $TMH = A(I_n)^{-1}$. As $TMH = \lambda^{-1}$, then lambda estimative is $\hat{\lambda} = I_n(A)^{-1}$. Now, the expected value of Y for $t \geq t_e$ (stable equilibrium point) is: $E(Y) = h(1-p) = A(\Delta t)^{-1}$ and, then, the estimation for p is given by the following equation:

$$\hat{p} = \frac{(h\Delta t - A)}{h\Delta t}. \tag{14}$$

This would be the correct estimation procedure for TMH – Equation (13), that is, knowing the curve Y in function of t, the value of A would be calculated by the Y integral between t_1 and t_2. Therefore, with the result of A (users x day) we can calculate the value of $E(Y)$ – average number of injured individuals in region R, who are taken to the hospital using U_s as a means of transport. In practice, the variation of the Y coefficient – relative dispersion of Y – is very small compared with h. This coefficient by definition is $c_Y = \sqrt{\dfrac{p}{h(1-p)}}$. Thus, c_Y in function of

A and Δt is given by:

$$c_Y = \sqrt{\frac{(h\Delta t - A)}{A\ h}}. \tag{15}$$

To clarify the developed methodology — estimations of TMH, $\hat{p}$, number of hospital beds n_L, $n_{L_{min}}$, TMF, TMH the three main capitals of states of the Northeast region of Brazil (region that presented the highest TMH) were chosen along with Belo Horizonte and Curitiba cities. For the estimations of TMH ($\widehat{TMH}$) it was considered that the relocation of the injured individual will be made to any hospital institution. In all calculations made for the minimum number of hospital beds $n_{L_{min}}$, the Equation (10) is utilized and for the considerations with reliability 100% ($\alpha = 1$). This result is compared with the effective beds with consideration to the missing beds estimation (deficit). All calculations above are represented in Table 2. In relation to the entry data for making the proper estimations, they were obtained from the Medical Sanitary Assistance – AMS (IBGE, 2005).

Table 2 - System User Population $\times$ Minimum Number of Beds with 100% Reliability

Cities	$Pop - h$	*Hospitalizations*-I_n	*Beds*-n_L	*Beds*-$n_{L_{mim}}$
Recife	1.486.869	328.870	8.089	8.448
Fortaleza	2.332.657	303.585	8.138	8.880
Salvador	2.631.831	279.623	7.676	8.026
B.Horizonte	2.350.504	384.838	8.719	9.002
Curitiba	1.727.010	388.392	6.013	6.323

Cities	TMF *(days)*	TMH *(days)*	$\hat{p}$	$(1-\hat{p})$
Recife	1.641,24	9,98	0,994560	0,00544
Fortaleza	2.794,77	9,78	0,996511	0,00349
Salvador	3.425,39	10,02	0,997083	0,00292
BeloHorizonte	2.221,13	8,27	0,996291	0,00371
Curitiba	1.617,35	5,65	0,996518	0,00348

The data of Table 2 was calculated from the estimated values of $p(\hat{p})$ and $(1-p)$ $((1-\hat{p}))$ of the cities of Recife, Fortaleza, Salvador, Belo Horizonte and Curitiba. This table also presents the total number of hospital beds represented by the private and official establishments (Municipal, State, Federal and partnered with SUS – Health Unic System).

For instance, the city of Salvador has a bigger population than Recife; but the probability of an individual being injured and relocated to hospitalization is $0,00292$ in Salvador city, while in Recife it is $0,00544$. That is why the minimum number of hospital beds required is greater than Recife (8.448) in comparison with Salvador (8.026). In regards to the estimated values of TMH, $n_{L_{min}}$ and TMF there are no abnormalities.

Observe that for the set of elements constituted by the estimated probabilities ($\hat{p}$), the value of the variance is of order of magnitude 10^{-7}. As the probabilities are situated in the same reliability range, it can be stated that the probabilities of an injured user being relocated to hospitalization is, in the statistical point of view, practically the same for all analyzed cities, no matter if it is a public or private institution.

Therefore, hospital beds can not be guaranteed to any individual from the five analyzed cities. Maybe these arduous deficits are caused by predominant factors, that hinder the increase of n_L. However, if the assistance policy is to guarantee hospitalization to the insured people of SUS, or to any individual in general – as predicted in the Federal Constitution – the hospitalization can be made in any institution – therefore, any variant for the estimation of TMH and p must be pondered with all hospitals.

For example, the hospital of the University of Pernambuco — "Hospital da Restauração" – HR – situated in one of the most heavily-trafficked avenues of Recife, has currently 535 beds. How many patients can utilize the hospital with 100% reliability? Applying the Equation (12), we have: $h_{max} = 83.397$ users. This way, this hospital can serve to an area with 84 thousand inhabitants. If the former is destined only to the SUS insured, this area will be bigger and, in any case, this area can be calculated by the estimation of the user density correspondent to the assistance policy adopted by the hospital.

To estimate p ($\hat{p}$) we must define the equations before and after the time for the stable equilibrium (t_e). Being h the population of region R with J non-injured individuals, then: $\lambda_j = (h-j)\lambda$ and $\mu_j = j\mu$, for $j = 0, 1, \ldots, h$. The average value of λ ($\overline{\lambda}$) can be calculated using the concept of weighted average. That is, $\overline{\lambda} = \sum_{j=0}^{h} \lambda_j P_j$, in which P_j represents the probability of an individual in state j not being a user of the system. Then,

$$\overline{\lambda} = \sum_{j=0}^{h} \lambda_j P_j = \sum_{j=0}^{h} \lambda(h-j)P_j. \tag{16}$$

Using Equation (1), for the calculation of P_j and making the proper substitutions, we arrive to the following expression for the average rate of injured individuals ($\overline{\lambda}$):

$$\overline{\lambda} = h\left(\frac{\lambda\mu}{\lambda+\mu}\right). \tag{17}$$

With regards to the average rate of non-injured individuals ($\overline{\mu}$), it is calculated using a procedure similar to the calculation of ($\overline{\lambda}$). Making the proper substitutions, we find the reduced form for $\overline{\mu}$.

$$\overline{\mu} = h\left(\frac{\mu^2}{\lambda+\mu}\right). \tag{18}$$

Thus, to define the average of p ($\overline{p}$), that is, the average probability of an individual being out of the system, the same formalized development for Equation (2) is utilized:

$$\overline{p} = \frac{1}{\left(1 + \frac{\mu}{\lambda}\right)}. \tag{19}$$

This way, it is determined that $\overline{p} = p$ – Equation (2). Thus, it is important for eliminating any doubts about the values of p_t to establish a function that explains the behavior of p_t with t. Being $q_t = (1 - p_t)$ the probabilistic distribution of the number of injured individuals in a period of time. Consider that $g(t)$ represents the probability density function (p.d.f.) of the distribution q_t, and $g(t)$ is the p.d.f. of the exponential distribution, represented below:

$$g(t) = (\lambda + \mu)\ e^{-(\lambda+\mu)t}, \quad t \geq 0 \quad and \quad g(t) = 0,\ t < 0.$$

To calculate $(1 - p_t)$, it is necessary to use the concept of cumulative distribution $G(t)$ for $g(t)$. For this, we must calculate the $g(t)$ integral in its domain.

$$G(t) = (\lambda + \mu)\ \int_0^t e^{-(\lambda+\mu)t} dt, \quad with\ t \geq 0.$$

Solving the integral in the interval of $(0,\ t)$, the expression that defines the probability of an individual suffering an accident before the stable equilibrium is found.

$$G(t) = -e^{-(\lambda+\mu)t}\Big|_0^t, \quad G(t) = 1 - e^{-(\lambda+\mu)t}. \tag{20}$$

As the probability of an individual suffering an accident after $t \geq t_e$ (stable equilibrium) is $(1 - p) = \left(\frac{\mu}{\lambda + \mu}\right)$, the probability of an individual suffering an accident in the period of time t is:

$$(1 - p_t) = (1 - p)\ G(t), \quad \text{or} \quad (1 - p_t) = \frac{\mu}{\lambda + \mu}(1 - e^{-(\lambda+\mu)t}\ ). \tag{21}$$

Consequently, the probability of an individual not suffering an accident in the period of time t is:

$$p_t = 1 - \frac{\mu}{\lambda + \mu}\left(1 - e^{-(\lambda+\mu)t}\ \right). \tag{22}$$

If X_t has a binomial distribution, then $E(X_t) = hp_t$ and $V(X_t) = hp_t(1 - p_t) = hp_t q_t$. The probability of existing j cured individuals – out of the system in time t is calculated through the Equation (3):

$$P(X_t = j) = C_h^j\ {p_t}^j\ (1 - p_t)^{h-j}, \quad j = 0, 1, \ldots, h. \tag{23}$$

After the stable equilibrium we get the limit of the Equation (22), that is, $\lim\limits_{t\to+\infty} p_t = \left(1 - \frac{\mu}{\lambda + \mu}\right) = p$. Thus:

$$\lim_{t\to+\infty} P(X_t = j) = C_h^j\ p^j\ (1 - p)^{h-j}, \quad j = 0, 1, \ldots, h. \tag{24}$$

The Equation (24) is invariant with t, because $\lim_{t \to +\infty} p_t = p$ and $\lim_{t \to +\infty} (1-p_t) = (1-p)$

This way, it is shown that the Equations (24) and (4) are equal; and that p represents the proportion of time an individual stays non-injured. Now, as Y_t is the number of system user individuals in time t, that are relocated to hospitals using the U_s, then, the probability of j injured individuals in time t also follows a binomial distribution.

$$P(Y_t = j \mid t \leq t_e) = C_h^j \ (1 - p_t)^j \ (p_t)^{h-j}, \quad j = 0, 1, \ldots h. \tag{25}$$

When t increases, p_t decreases from 1 to $p = \left(\dfrac{\lambda}{\lambda + \mu}\right)$, remembering that the initial condition equivalent to a $p_0 = 1$; $(1 - p_t)$ increases from 0 to $\left(\dfrac{\mu}{\lambda + \mu}\right)$, when t increases from 0 to $+\infty$. In actuality, when $t \geq t_e$, we can consider p_t and $(1 - p_t)$ as constants and, consequently, the value of $E(Y_t)$ is, too, constant. This way, we must do a detailed evaluation for the dimension of t_e. The evaluation for t_e is done based on the data of the $TMHs$ calculated previously for the five cities: Recife, Fortaleza, Salvador, Belo Horizonte and Curitiba. These estimated periods of time $TMHs$ for the cities are, respectively: 8,98 days, 9,78 days, 10,02 days, 8,27 days and 5,65 days.

Substituting now the expression $(TMF^{-1} + TMH^{-1})$ by $(\mu + \lambda)$ and later calculating for each city the function $G(t) = 1 - e^{-(\lambda+\mu)t}$, with t varying between 0 and $+\infty$, we arrive to the equilibrium when the distribution function $G(t) = 1$, that is, when the result of the expression $(1 - e^{-(\lambda+\mu)t})$ is equal to 1. In Table 3, the results of t_es calculated for each city are presented, considering the value of the distribution function $G(t) = 1 - e^{-(\lambda+\mu)t} = 1$.

Table 3 - Values of Time for the Stable Equilibrium – t_e

City	TMH(days)	t_e(days)	TMF(days)	%Equil.
Recife	8,98	110	1.641	6,70
Fortaleza	9,78	130	2.795	4,60
Salvador	10,02	130	3.425	3,80
Belo Horizonte	8,27	110	2.221	4,90
Curitiba	5,65	70	1.617	4,30

The values of the t_es vary from 70 days to 130 days. Curitiba has the shortest time and Fortaleza the longest, which is tied with Salvador. Recife is equal to Belo Horizonte, with a $t_e = 110$ days. Calculating the percentages of participation of the t_e with the respective $TMFs$, we come to a variation of 3,8% (Salvador) to 6,70% (Recife). The average of these percentages of participation is 4,86% with a standard deviation of $0,96\%$ and the dispersion coefficient $0,19$ from the average.

In the case of regions with shorter TMH, the stable equilibrium t_e is reached faster. This way, it is viable to establish conditions, in the sense of defining a strategy for the values of p before and after the stable equilibrium. In Cordeiro (2012), a detailed behavior analysis of the curve $E(Y_t)$ is made with the intention of making a more robust estimation of p in the period of time. The obtained results suggest a difference in the average percentage of the relative error varying from $0,06743\%$ to $0,07106\%$. However, when crossing the averages of the values of $\hat{p}s$ there is a difference, which order of magnitude is 10^{-6}.

4 Random variable time between successive entries in hospitalization – $TEESH$

The time between successive entries in hospitalization ($TEESH$) is a concept that is independent of how the patient got to the hospital, by using U_s or not. But the time between successive entries in emergency ($TEESE$) for patients that need emergency treatment must have the transport service guaranteed – pre-hospital assistance via a U_s. This provided service satisfying serves to all emergency patients, who are its potential users.

Thus, the other patients enter hospitalization via queues, with elaborated guidelines, being booked for hospitalization, without emergency. To dimension the number of U_s capable of serving all injured individuals in region R, that are hospitalized with emergency, it is important to define a variable that explains the time between successive entries for these potential users; patients that arrive via pre-hospital mobile care denominated U_s. Initially, this variable – time between successive entries for patients that arrive via U_s ($TEESU_s$) is equal to $TEESE$.

Considering that the region reached the stable equilibrium and being X the number of non-injured individuals, then the average value of this time conditioned to j cured individuals is: $E(TEESH)|\ j$ cured) $= E(TEESH(j))$. Therefore, the average value $E(TEESH)$ is calculated weighting the $E(TEESH)|\ j$ cured) with the P_j:

$$E(TEESH) = \sum_{j=0}^{h} E(TEESH(j))P_j. \tag{26}$$

Observe that in a given region R of population h in time t has j cured individuals equal to $E(X) = hp$ e $(h - j)$ and injured equal to $E(Y) = h(1 - p)$. TNA is the variable that measured the time of a non-injured individual out of the system. This way, for hp cured individuals:

$$TEESH = MIN(TNA, TNA, \dots, TNA) \quad \text{or} \quad TEESH = MIN(TNA)^{hp}.$$

As the $P(TNA \geq t) = \left(1 - \int_0^t f_{TNA}\ dt\right) = \left(1 - \int_0^t \mu e^{-\mu t} dt\right) = e^{-\mu t}$, then:

$$P(TEESH(hp) \geq t) = e^{-\mu hpt}. \tag{27}$$

Thus, there are always hp cured individuals susceptible to accidents. Therefore, the p.d.f. of the variable $TEESH$ is exponential, because the $TEESH(hp)$ is exponential. Considering that I_n represents the number of hospitalizations in the period of $\Delta t = 1$ year, then $I_n = \mu hp\Delta t$; as $\mu p = \lambda(1-p)$, then $\left(\frac{I_n}{\Delta t}\right) = \lambda h(1-p)$ and being A the area below the curve Y(t) – Figure 1, then $h(1-p) = \left(\frac{A}{\Delta t}\right)$, which is the Equation (14) for the calculation of p.

For the ZAs of the U_s it is of good measure to affix to h an order of magnitude that varies between 10^4 and 10^5 inhabitants. This way, it is possible to decentralize the U_s or as a single region to concentrate all U_s in a single spot. Distributing them in assistance zones is a more viable solution both economically and efficiently in relation to the response time. This way, the travel times computed by the U_s are shorter with the decentralization, when compared to the travel times of the U_s concentrated in a single spot (GOLDBERG *et al.*, 1990).

A structure equation, similar to Equation (27) – $P(TEESH(hp) \geq t) = e^{-\mu hpt}$, can be obtained considering the proportion of the $TEESH$ in relation to Δt, that means, $\left(1 - \frac{t}{\Delta t}\right)$. This way, the $P(TEESH(hp) \geq t)$ can be calculated also through the equation below:

$$P(TEESH(hp) > t) = \left(1 - \frac{t}{\Delta t}\right)^{I_n},$$

being $I_n = \mu hp\Delta t$. As I_n – hospitalizations or entries of injured individuals in the hospitals – is very big because the period Δt is long (1 year), then $P(TEESH(hp) \geq t)$ is approximated to exponential. Remember that $lim_{x\to+\infty}\left(1 - \frac{1}{x}\right)^x = e^{-1}$. So,

$$P(TEESH(hp) > t) = \left(1 - \frac{t}{\Delta t}\right)^{I_n} = e^{\left(-\frac{t}{\Delta t}\right)I_n} \text{ or } P(TEESH(hp) > t) = e^{-\mu hpt}.$$

Differentiating in relation to t, the distribution function accumulated from the variable $TEESH$, we obtain the p.d.f. $f_{TEESH}(t)$ (remembering that I_n is big),

$$f_{TEESH}(t) = \left(\frac{I_n}{\Delta t}\right)\left(1 - \frac{t}{\Delta t}\right)^{I_n-1} \text{ or } f_{TEESH}(t) = \mu hp\left(1 - \frac{\mu hpt}{I_n}\right)^{I_n-1}. \quad (28)$$

The statistics $E(TEESH)$ and $V(TEESH)$ from this distribution $f_{TEESH}(t)$ are similar to the Equation (27). As I_n is big, then the relation $\left(\frac{I_n}{I_n+1}\right) \approx 1$. We have,

$$E(TEESH) = \frac{\Delta t}{I_n+1} \text{ and } V(TEESH) = \frac{I_n}{(I_n+1)^2(I_n+2)}\Delta t^2. \quad (29)$$

When replacing $\left(\frac{I_n}{\Delta t}\right)$ for μhp, we obtain the expected value $E(TESEH) = (\mu hp)^{-1}$, which is the same found from $E(TEESH)$ of Equation (27). In the long run, $\Delta t = 1$

year, the distribution processes of $TEESH$ are coincidental. Actually, the expected value $E(TEESH)$ is greater for the exponential – most favorable case –; on the good side, the standard deviation is greater.

Based on the average of the $I_n s$ related to the five cities, the imputed values for the increase percentages are: $\%\Delta(E(TEESH)) = 0,0002973\%$ and $\%\Delta\sigma_{TEESH} = 0,0004450\%$. This way, working for the distribution of the variable $TEESH$ as exponential – Equation (27) is closer to reality and, therefore, its sudden variations are predicted, resulting from a higher standard deviation.

5 Studies of the random variables that represent the time between the entries in hospitalization $(TEESHU_s)$ and in emergency $(TEESE)$

Consider the random variable that represents the time between successive entries for injured individuals that need emergency treatment – $TEESE$. Supposing I_n entries in hospitalization during the period Δt. Being K entries in emergencies then $(I_n - K)$ represents the entries in the hospitals without emergency. For the hypothesis based on a service policy for the pre-hospital assistance, the equation $TEESE = TEESHU_s$, is feasible, in which the $TEESHU_s$ represents the time between successive entries in hospitalization for the injured that need a mobile pre-hospital assistance – U_s. It is very hard to determine $TEESHU_s$ beforehand, from statistics made for the entries with the service units (U_s), because there is no availability to quantify the data relative to the number of emergency calls that were not answered and effected to SAMU-192. The $(I_n–K)$ entries in the hospitals are statistics of easy determination and more reliability, because they can be made through the hospital inpatient admission order. Therefore, the hypothesis of $TBESE = TEESHU_s$ is the most unfavorable case of assistance through U_s, that is, assisting every emergency accidents.

Two types of failure can occur in the assistance given by the service units. The first comes from the lack of assistance by the U_s to the injured, that is, the injured dies before receiving medical assistance by the U_s. On the second type, the injured is assisted by the U_s, but dies on the way to the hospital – dies before being hospitalized. It is aggravating that there is no statistics from SAMU that makes it possible to estimate the failure occurrence percentages.

The assistance requests through a U_s in which the injured dies before arriving for hospitalization are inserted into the K emergencies. By exclusion, Z_1 is the variable that represents the number of entries with emergency that are not realized, via U_s, although they are done more efficiently via other means of transport. This can occur sometimes, because the patient's transportation is done more quickly by a vehicle that is closer to him/her – private vehicle.

On the other hand, Z_2 also exists, which represents the number of entries without emergency in the hospitals, that due to the patient's health condition, the transportation must be done by a U_s, although the time of the transportation can be booked beforehand.

In general, the values of Z_1 and Z_2 are negligible when compared to I_n and K; this way, we can suppose that $Z_1 = Z_2$, making the equation $TEESE = TEESHU_s$ valid, and consequently, the expected values as well: $E(TEESE) = E(TEESHU_s)$.

Considering that the K entries in emergency are independent and distributed in a random way at any moment, in a period Δt and, being both probable, that **one entry** can be in any point of Δt, then we can establish for the expected value $E(TEESE) = \left(\frac{1 \times \Delta t}{K+1}\right)$ analogous to the Equation (29), that is to say, $E(TEESHU_s) = \frac{\Delta t}{I_n + 1}$. Therefore, crossing the equation $E(TEESE)$ with Equation (29) comes Equation (30).

$$E(TEESE) = \left(\frac{I_n + 1}{K+1}\right) E(TEESH). \tag{30}$$

As a result of the similarity of the equations mentioned, we can state that the process of distribution of K emergencies in the I_n entries in hospitals – injured individuals that arrive to the hospitals – spaced out from $E(TEESH)$ (discreet process) is equivalent to considering that the emergency entries are independent and randomly distributed in a period of time Δt (continuous process). Therefore, it is important to state that the random variable $TEESE$ has a distribution analogous to the random variable $TEESH$.

This way, we can make for the equations $P(TEESE)$, f_{TEESE} and $V(TEESE)$ from distribution $TEESE$, expressions analogous to the expressions (28) and (29) for the distribution $TEESH$. Therefore, to establish these equations we only need to change the variable I_n for the variable K:

$$P(TEESE > t) = \left(1 - \frac{t}{\Delta t}\right)^{K} \quad \text{and} \quad f_{teese}(t) = \frac{K}{\Delta t}\left(1 - \frac{t}{\Delta t}\right)^{K-1}. \tag{31}$$

Thefore,

$$E(TEESE) = \left(\frac{\Delta t}{K+1}\right) \quad \text{and} \quad V(TEESE) = \frac{K}{(K+1)^2(K+2)}\Delta t^2. \tag{32}$$

Consider that $\frac{\psi}{100}$ represents the proportion of entries, which purpose is the emergency assistance. As $\Delta t = \frac{I_n}{\mu hp}$ and $K = \left(\frac{\psi}{100}\right) I_n$, it occurs that:

$$\begin{aligned} &P(TEESE > t) = \left(1 - \frac{t}{\Delta t}\right)^{\left(\frac{\psi \mu hp \Delta t}{100}\right)} \quad \text{or} \\ &P(TEESE > t) = \exp\left(-\frac{\psi \mu hpt.}{100}\right). \end{aligned} \tag{33}$$

As verified before, the exponential represents an excellent approximation for the variable $TEESH$ and also for the variable $TEESE$, whose average is $\dfrac{100}{\psi\mu hp}$. This approximation is favorable, because it imputes in the long run $\Delta t = 1$ year a more expected value for $E(TEESE)$:

$$E(TEESE) = \left(\frac{\Delta t}{K+1}\right) \quad \text{or} \quad E(TEESE) = \left(\frac{K}{K+1}\right)\frac{100}{\psi\mu hp}. \tag{34}$$

In a very long Δt period, $\left(\dfrac{K}{K+1}\right) \approx 1$. This way, the Equation (34) is in function of the statistic ψ and the entries in hospitals μhp.

For the exposed reasons, the adoption of the exponential distribution, also, for the variable $TEESE$ is more appropriate, because it provokes less risks derived from a higher variation for the expected value, that is $\%\Delta(E(TEESE)) = \dfrac{1}{K}$. However for the variable $TEESE$, the number of emergency entries in a long period of time $\Delta t > 0$ also follows a Poisson distribution $\left(\dfrac{\psi}{100}\mu hp\Delta t\right)$. It is known that $K \sim P\left(\dfrac{\psi}{100}\mu hp\right)$, then the probability of the hp cured individuals having k hospitalized individuals in emergency in Δt is:

$$P(K(\Delta t) = k) = \frac{\exp\left(-\dfrac{\psi\mu hp\Delta t}{100}\right)\left(\dfrac{\psi\mu hp\Delta t}{100}\right)^k}{k!} \quad if \quad \Delta t > 0. \tag{35}$$

and $\quad k = 0, 1, 2, \ldots$

This way, we have the mathematical formalization of the variable distributions of TMF, $TEHS$, $TEESH$ and $TEESE$, as well as their respective statistics, all inherent to the hospitals. These hospitals will be treated as integration stations, whose means of transport are the service units $U_s s$, for the pre-hospital assistance. All in all, to compute all statistics, it is fundamental to safely estimate the variable ψ, that represents the proportion of the entries that are destined strictly to hospital emergencies.

5.1 Study of the variable ψ – Proportion of entries destined to hospital emergencies

There are some difficulties on the construction of the variable ψ. The first one refers to the data collection relative to strict emergency; since the last statistical yearbook made by (IBGE, 2005) does not contain any information for this purpose. The second is that in hospitals, the hospital inpatient admission order reflects in its majority normal entries. The emergency entries established in the medical record receive a mark of adequate or inadequate characterization. Although the variable ψ can easily be superiorly limited – security factor favorable for the dimensioning of the U_s – its big imprecision can generate big idleness and majority encumber the costs associated to transportation. That is because an improvement in the U_s service can significantly increase the number of **emergency entries**.

To define the estimation for ψ in favor of security, we use the law of large numbers. The estimation is based on the study developed about "The profile of emergency in the "Hospital da Restauração" – HR: an analysis of the possible impacts after the municipalization of health services", (CAVALCANTI and ARAÚJO, 2004). The authors emphasize that between 1993 and 2001 the number of emergency entries increased almost 50%. The traumatology procedure participates with 35%, probably in its vast majority caused by traffic accidents.

Almost all patients of HR (90%) come from Recife or from the Metropolitan Region, being Monday and daytime the periods with highest demand. The HR is characterized as an emergency hospital, with a monthly average of 25.800 entries being 12.000 ($46,51\%$) strictly emergencies. Not including patients diagnosed with cholelithiasis submitted to operation on emergences (CUNHA *et al.*, 2016). What is most surprising in the assistance point of view is that in the computed entries destined to HR's emergency, $74,5\%$ are characterized as inadequate for this emergency – patients whose diagnoses could be cured by basic treatment. (SALLA *et al.*, 2002) found in a broad hospital research a $60,6\%$ percentage of entries in emergencies that were characterized as inadequate.

In a survey made in the 62.500 entries in HR – about 73 days, in each 2 emergencies of 100 observed entries, in almost 99% of them, the relative frequency observed $\left(\dfrac{K}{I_n}\right)$ would be at less than 2% from $46,51\%$. It can be claimed that $\hat{\psi} = 48,51\%$ is a safe estimation for a hospital with a strict emergency characteristic. This way, we observe for $\hat{\psi}$ the variation interval $(44,51\%,\ 48,51\%)$.

Based on the calculations already properly computed, it is verified that in Recife, in the year of 2005, the expected value $E(TEESE) = 9,32$ minutes, that is to say, averagely in each 9,32 minutes an injured individual is entered in emergency. Furthermore, the probability that the $TEESE$ in 2005 is superior to 10 minutes is of $0,007821$. It can be observed that the tendency of the expected value for the variable $TEESE$ is to reduce substantially with the increase of h; particularly for constant ψ.

6 Study of the service time variable TS of Any U_s

The care policy for the service units (U_s) is to assist only emergency patients. Therefore, considering Z_1 – the number of entries in emergency not carried out by U_s, and Z_2 – the number of non-emergency entries using the U_s, to maximize the emergency care via U_s we must look for $Z_1 = Z_2 = 0$. This way this situation converges to the most unfavorable case for the dimensioning of the U_s aimed at pre-hospital care. This way, it can be considered that the variable Time Between Successive Entries in Hospitalization Using Service Unit U_s ($TEESHU_s$) represents the same variable $TEESE$. So the equations for the variable $TEESHU_s$ are the same defined for $TEESE$. It is worth to emphasize that the values of Z_1 and Z_2 participate with very small representativeness, when compared to the volume of I_n – entries in hospitals and K – entries in emergency. Therefore it is appropriate to set the U_s only to care for emergencies, and this way we can consider $Z_1 \simeq Z_2 \simeq 0$.

Being TS a random variable that establishes the service time of any U_s and W, an also random variable that represents the wait time for a U_s to be available and be assigned to the call (wait time in queue). Initially, the variable TS is defined as the sum of a set of random independent variables defined below:

- TP: Time spent for the U_s to leave; that is, the time between the occupation of the U_s and its exit from the storage location;
- TV: Travel time of departure or arrival;
- TA: Time spent for picking up the injured patient and helping him on spot, that is, time spent between the arrival to the location of the accident and the departure of the U_s from the location;
- TD: Time spent to deliver the patient to the hospital until the clearance of the U_s – service unit ready for return.

Besides those, the variable TS is defined by the sum below:

$$TS = TP + 2TV + TA + TD, \tag{36}$$

he variables TP, TV, TP and TD depend a lot on the work dynamic established for the collection and confirmation of the results. As a consequence of that, the estimations produced can provoke big distortions, especially because of the strong sensitivity of these variables. This way, the estimations to be obtained are originated from a probabilistic distribution properly adjusted to the observed statistics. A feasible distribution to be adjusted to a specific interval of time Δt is the beta distribution.

Suppose, the following time measurements, all obtained from a sample of Δt: $\overline{\Delta t}$ (average), Δt_{min}, Δt_{max} and $\sigma_{\Delta t}$ (standard deviation). Consider the following distribution for Ω defined by the equation below:

$$\Omega = \frac{\Delta t - \Delta t_{min}}{\Delta t_{max} - \Delta t_{min}}. \tag{37}$$

Exemplifying, if Δt corresponds to the sequence $(t_1, t_2, t_3, t_4, t_5)$, the assumed values for Ω are in between $0 \leq \Omega \leq 1$. This is valid for any sequence. Therefore, it is possible to consider that $\Omega \sim \beta(a, b)$, where the a and b are real positive parameters of the distribution denominated scale and declivity, respectively. These parameters are determined from the equations below:

$$\overline{\Omega} = \frac{\overline{\Delta t} - \Delta t_{min}}{\Delta t_{max} - \Delta t_{min}} = \frac{a}{a+b} \text{ and } \sigma_\Omega = \frac{\sigma_{\Delta t}}{\Delta t_{max} - \Delta t_{min}} = \frac{1}{a+b}\sqrt{\frac{ab}{a+b+1}}. \tag{38}$$

With relation to the variable TV, it depends on the geometry of the assistance service zone (ZA) made by the service unit (U_s) and also of the location where the hospital is. As the clearance of the U_s obey a queueing system, it is possible to calculate through the queueing model the average and variance of the variable TS. The response time TR, as defined previously, represents the time between the U_s request and its arrival at the patient, This variable is represented by the following equation:

$$TR = W + TP + TV. \tag{39}$$

After the definitions of the equations for the variables TS and TR and, based on the distribution that represents the variable $TEESE$, we seek for a representation for the variable distribution that represents the time between the successive requests of U_s in the screening center for assistance to the injured – $TECSU_s$.

7 Study of the variable time between successive request of $U_s - TECSU_s$

For the purpose of studying the distributions for the variables $TECSU_s$ and $TEESHU_s$ we must consider a hospital that is assisting a small user population situated in a determined ZA, whose dimension reproduces a smaller TR. Thus, the number of service units n_s, allocated to the hospital tends to be small. For example, $n_s = 1$ to $n_s = 3$. It is important to emphasize that to maximize the care system's efficiency, it is necessary to increase the idleness factor (1-ρ) or to decrease the usage ρ; or to change the wait time in queue to $W = 0$ and consequently the following inequations must be obeyed, that is $TS < TECSU_s$ and $TS < TEESE$. The equation that measures this idleness is expressed by the usage factor ρ, being determined in function of the variables TS, $TECSU_s$ and n_s – number of service units, according to the expression below:

$$\rho = \frac{E(TS)}{E(TECSU_s)\ n_s}. \tag{40}$$

As seen before, the distribution of the variable $TEESE$ is recognized and represented by the Poisson distribution. This way, the behavior of the system will be represented by the variable $TECSU_s$ – system entry, by the variable TS – service assistance, and by the variable $TEESE$ – system exit. Being the system exit a Poisson distribution and the variable TS ruled by the exponential distribution, then the variable $TECSU_s$ follows a Poisson with the same exit rate. Therefore, we have an stigmatic system and, this way, we can state that $TECSU_s = TEESE$. A detailed demonstration about this theme is found in (HILLIER *et al.*, 1986). This way we stumble over a problem that is based on the strict queueing theory. This queueing system is characterized by being a Markov process. Still in relation to the queueing system, the moments (expected value and variance) of the chosen distributions $TECSU_s$, $TEESE$ and TS are extremely necessary as an optimization tool. In case of finding difficulties in the expression of the moments of these distributions, it is necessary to find approximated methods for the acquisition of the moments, as for example, the Laplace Transform. Making the response time exponentially distributed to a queueing system of type $M/M/n_s$, then the probability distribution function of the variable TR is given by:

$$P(TR \leq t) = F_{TR}(t) = 1 - e^{-\mu \; n_s(1-\rho)t}. \tag{41}$$

Observe that the minimization of the response time is caused due to the increase of the idleness factor $(1 - \rho)$ or usage minimization. In the same way, it can be demonstrated that for the system $M/M/n_s$ the probability distribution of the random variable W – wait time is represented by the equation:

$$P(W \leq t) = F_W(t) = 1 - e^{-\lambda \; n_s(1-\rho)t}. \tag{42}$$

This way it is shown that the idleness increase or the usage decrease of the U_s responds sensitively to the wait time reduction W and also to the behavior change of the response time function (TR); this time rapidly decreases with the increase of the idleness. To demonstrate that these are exponential times, it is necessary to apply the Laplace Transform to $f_W(t)$ e $f_{TR}(t)$. It is shown that $F_W(s) = L[\lambda e^{-\lambda t}]$, having as exponential the $f_W(t)$. The same procedure for the response time, that is $F_{TR}(s) = L[\mu e^{-\mu t}]$, having f_{TR} as exponential, too.

This way, it can be generalized for queuing system with assistance through U_s, the representation $M/G/n_s$, where the M represents the Poisson's entry distribution, G a general distribution for the service time TS and n_s the number of units U_s, that represent the service servers. Therefore, it is possible to admit for this general system, that the entry distribution is equal to the exit distribution; therefore, $TECSU_s = TEESE$. It can be observed that for the variable distribution $TECSU_s$ the Equations (31) and (35) explain this distribution.

It is worth to emphasize that there are particular difficulties in adopting the same distribution $TECSU_s$ throughout the entire day. These difficulties are inherent to the abrupt variations that occur in the demand – higher search of the U_s at rush hours – periods with larger traffic volumes. This way, it is possible to partition the day in periods of time. For example, the $TECSU_{s_1}$ of smaller value and constant for the period $\Delta t_1 = n_d P_1$; the $TECSU_{s_2}$ of greater value and constant for period $\Delta t_2 = n_d P_2$. Being n_d the number of days in $\Delta t = \Delta t_1 + \Delta t_2$ and P_1 two periods of the day defined as the first shift from 6:00H to 14:00H and the second shift from 14:00H to 22:00H. While the second period P_2 refers to the third shift from 22:00H to 6:00H. Therefore, $\Delta t_1 = 2\Delta t_2$ e $P_1 = 2P_2$. In this sense, there is the necessity to guarantee a homoscedasticity of the request rates during the study period, in such a way that guarantees the Poisson process.

8 Methodology for estimating of the request rates of the U_s – $\overline{\lambda_1}$ and $\overline{\lambda_2}$

Consider that the request rate $\overline{\lambda_1}s$ is constant in Δt_1 and $\overline{\lambda_2}s$ is constant in Δt_2. Suppose that in the first period Δt_1 all the consecutive P_1s are formed and

later in the second period Δt_2, all the consecutive $P_2 s$ are also formed. The request rates $\overline{\lambda_1}$ and $\overline{\lambda_2}$ of the U_s in the independent process and identically distributed over Δt_1 and Δt_2, respectively and, admitting that $TECSU_S$, $TECSU_{s_1}$ and $TECSU_{s_2}$ are exponentials.
As the variable K represents the number of emergency entries using as means of transport the U_s, and making ℓ the request probability of this U_s being in Δt_1, the expected values for the variables $TECSU_s$, $TECSU_{s_1}$ and $TECSU_{s_2}$ are:

$$E(TECSU_s) = \frac{\Delta t}{K},\ E(TECSU_{s_1}) = \frac{\Delta t_1}{\ell K} \text{ and } E(TECSU_{s_2}) = \frac{\Delta t_2}{(1-\ell)K}. \quad (43)$$

Having considered $\Delta t_1 = \frac{2}{3}\Delta t$ and $\Delta t_2 = \frac{1}{3}\Delta t$, that is, in K hospitalizations with service units, it is seen that ℓK hospitalizations are at day and $(1-\ell)K$ at night. Thus, making the proper substitutions in the previous equations:

$$E(TECSU_{s_1}) = \frac{2}{3\ell}E(TECSU_s) \text{ and } E(TECSU_{s_2}) = \frac{1}{3(1-\ell)}E(TECSU_s). \quad (44)$$

Observe that $E(TECSU_s) = \frac{100}{\psi\mu hp}$ and $V(TECSU_s) = \frac{10000}{(\psi\mu hp)^2}$. Therefore, it is possible to put $E(TECSU_{s_1})$ and $E(TECSU_{s_2})$ in function of $\frac{100}{\psi\mu hp}$. What happens is:

$$E(TECSU_{s_1}) = \frac{2}{3\ell}\frac{100}{\psi\mu hp} \quad \text{and} \quad E(TECSU_{s_2}) = \frac{1}{3(1-\ell)}\frac{100}{\psi\mu hp}. \quad (45)$$

After the expressions of the expected values of the variables $TECSU_{s_1}$ and $TECSU_{s_2}$ we come to the result of the variances and with that the distributions' characteristics:

$$V(TECSU_{s_1}) = \frac{40000}{(3\ell\psi\mu hp)^2} \quad \text{and} \quad V(TECSU_{s_2}) = \frac{10000}{[3(1-\ell)\psi\mu hp]^2}. \quad (46)$$

The variables $TECSU_{s_1}$ and $TECSU_{s_2}$ have exponential distributions of parameters $\overline{\lambda_1}$ and $\overline{\lambda_2}$. In all explaining equations of the variables $TECSU_{s_1}$ and $TECSU_{s_2}$ it is necessary to estimate the value of the proportion ℓ. The methodology for this estimation is similar to the one described for the variable ψ. Therefore the estimated values of the average rates for the attendances in Δt_1 and Δt_2 are:

$$\overline{\lambda_1} = \frac{3\ell\psi\mu hp}{200} \quad \text{and} \quad \overline{\lambda_2} = \frac{3(1-\ell)\psi\mu hp}{100}. \quad (47)$$

9 Dimensioning of the U_s to efficiently help the users of an assistance zone ZA

In Cordeiro (2012), it is demonstrated that for each station of an service unit (U_s) that belongs to an assistance zone we must use the queuing system structured in the form of $M/G/n_s$ (∞, First in First out $-FIFO$) with entries (λs) given by the Equations (47):

$$\overline{\lambda_1^*} = \frac{3\ell\psi\mu hp(1-lost_1)}{20000} \quad \text{and} \quad \overline{\lambda_2^*} = \frac{3(1-\ell)\psi\mu hp(1-lost_2)}{10000}.$$

The service time – Equation (36) given by $TS = TP + 2TV + TA + TD$. All those times are estimated from a function adjusted to the observed values and $E(TV)$ and $V(TV)$ are determined by the equation – $E(D) = 0,50\sqrt{A}$ and $V(D) = \frac{A}{36}$. Depending on the layout of the ZA, $V(D) = 0,108\frac{A}{N}$ (N, number of sub-zones of the ZA). More details consult (CORDEIRO, 2015).

$\overline{W_D}$, $\overline{W_M}$ and $\overline{W}$ are the average wait time in the queue of an assistance request of an injured individual using U_s, considering the queuing models $M/D/n_s$, $M/M/n_s$ and $M/G/n_s$, respectively. Owen (1971) while studying the average wait time in queue, in the $M/E_m/n_s$ model through theoretical results obtained with simulation, concluded that it is possible to acquire an adequate approximation for this $\overline{W}$, from the interpolation between $\overline{W_M}$ and $\overline{W_D}$, as long as the average wait time in queue is small ($\overline{W} \approx 0$).

This way, it is possible to consider for the queuing system the model $M/E_m/n_s$ adjusted to the general model $M/G/n_s$ and, as a consequence of that, obtain the $\overline{W}$ interpolating between $\overline{W_M}$, $\overline{W_D}$ by the means of:

$$\overline{W} = \overline{W_D} + c^2(\overline{W_M} - \overline{W_D}) \quad \text{and} \quad c = \frac{\sigma_{TS}}{E(TS)}. \tag{48}$$

Remembering that $TS \sim \beta(a,b)$, then $c = \frac{1}{a}\sqrt{\frac{ab}{a+b+1}}$. The probability density function of the Erlang of order m with request rate (call rate) of U_s constant λ is:

$$f(x) = \frac{\lambda^m x^{m-1} e^{-\lambda x}}{(m-1)!} \quad 0 \leq x < \infty. \tag{49}$$

The expected value of X is given by:$E(X) = \frac{m}{\lambda}$ and the variance, by: $V(X) = \frac{m}{\lambda^2}$. For $m = 1$, we have the exponential distribution and a Poisson arrival process. As m increases, the distribution's relative dispersion decreases, reaching the deterministic situation (constant intervals between requests) when $m \to \infty$. As $m = 1, 2, \ldots n_s$, then what shall be the probability of all n_s being occupied – a U_s request to enter the queue or the probability of the number of injured users being greater or equal to n_s? It is used, in this case, the Erlang's delay formula, where $P(n_s, a)$ is equal to this probability with $a = \frac{\overline{\lambda}}{\overline{\mu}}$, that represents the number of U_s requests per service time. Based on the usage factor (utilization factor) $\rho = \frac{\overline{\lambda}}{\overline{\mu} n_s}$ and on the idleness factor $(1-\rho)$, we have Erlang's delay formula given by the equation:

$$P(n_s, a) = \frac{a^{n_s}}{[(n_s-1)!(n_s-a)]\sum_{m=0}^{(n_s-1)} \frac{a^m}{m!} + a^{n_s}} \qquad 0 < a < n_s. \tag{50}$$

For the calculation of the average wait time in queue $\overline{W_M}$ for the model $M/M/n_s$ we need to divide the $P(n_s, a)$ by the difference between the attendance total and the average request rate of the U_s:

$$\overline{W_M} = \frac{P(n_s, a)}{(\overline{\mu} n_s - \overline{\lambda})}, \quad \text{when} \quad a < n_s. \tag{51}$$

To express the average wait time in queue $\overline{W_D}$ for the model $M/D/n_s$, we use the Molina's approximated formula (CORDEIRO, 2012), if $n_s \geq 1$:

$$\overline{W_D} = \frac{P(n_s, a)}{2(\overline{\mu} n_s - \overline{\lambda})}, \quad \text{when} \quad n_s = 1. \tag{52}$$

$$\overline{W_D} = \frac{P(n_s, a)}{(\overline{\mu} n_s - \overline{\lambda})} \frac{n_s}{n_s + 1} \frac{1 - \left(\dfrac{a}{n_s}\right)^{n_s+1}}{1 - \left(\dfrac{a}{n_s}\right)^{n_s}}, \quad \text{when} \quad n_s > 1. \tag{53}$$

The term $P(n_s, a)$ that represents the probability of all n_s being occupied is found in the Erlang distribution's table. The purpose of illustration – for the case study addressed – the "Hospital da Restauração" – HR – located in the Agamenon Magalhães avenue (Recife), assists all emergency injured. The number of beds n_L is equal to 535, data from (IBGE, 2005). For the city of Recife, we have the following information: a) Average estimated time in hospital: $\widehat{TMH} = 8,98$ days; b) Average estimated departure rate from the hospital $\hat{\mu} = 0,0006092953$; c) The estimated probability of an individual suffering an accident $(1 - \hat{p}) = 0,00544$; d) Average estimated proportion of entries destined strictly to emergency $\hat{\psi} = 0,4851$. The estimated request probability of U_s (in Δt_2 – night period) is $(1 - \hat{\ell}) = 0,1934$ and in Δt_1 (day period) is $\hat{\ell} = 0,8066$. This hospital is located in a ZA located in the Politic Administrative Center Region – $RPA1$.

This RPA presents a population of 77.607 inhabitants (IBGE, 2005), distributed through an area of 1.606 ha (16,06 Km^2) and 22.579 residences, with population density $\dfrac{48,63 \text{ hab}}{ha}$, equivalent to $\dfrac{4.832 \text{ hab}}{Km^2}$. As all the inhabitants are part of the SUS, when injured they must use as emergency the HR. Furthermore, it is admitted that all the U_s are destined exclusively to emergencies. For an assistance of good quality, that is, a good service provided by the pre-hospital mobile care system, it is considered for the Maximum Response Time (TMR): $\widehat{TMR} = 7$ min (approximately 50% more than the established for the city of New York). Furthermore, because it is a hospital located in a perimeter of a great flux of vehicles, it is considered an estimated speed of $\hat{v} = 35\ Km/h$ and the estimated probability of finding an available bed is $\alpha = 0,975$, that is, from the normal distribution table, the function $\Phi^{-1}(\alpha) = 1,96$. Reporting the Equation (11), for the calculation of the user population h with $\alpha < 1$, we have $h = 91.047$users.

Considering the constant population density $d = \dfrac{4.832 \ \ hab}{Km^2}$, it happens that the coverage area of this hospital is estimated to $\hat{A} = \dfrac{91.593}{4.832} Km^2 = 18,85\ Km^2$. So we have to define a rule to make the knowledge of these variables tangible via their estimations, because they are important for the acquisition of the service time (TS) and of the response time (TR). Consider that these variables are evenly distributed over the 24 hour period (BURNS *et al.*, 1985), inside the following variation intervals in minutes: $TP \sim U(0,4)$, $TA \sim U(0,8)$ and $TD \sim U(0,4)$. So, $E(TP) = 2\ min$, $E(TA) = 4\ min$ and $E(TD) = 2\ min$. The variances are: $V(TP) = 1,33\ min^2$, $V(TA) = 5,33\ min^2$ and $V(TD) = 1,33\ min^2$. For the characterization of the assistance zone, consider that ZA has geometrical shape different from the square with spin of $\angle 45$ degrees in relation to the horizontal axis. Thus, to analyze the compatibility of ZA, it is necessary to verify if the equations below demonstrated in (CORDEIRO, 2012) are satisfied, that is:

$$A \leq [TMR - E(TP)]^2 \frac{v^2}{1410} = 21,72 Km^2.$$

As the coverage area is estimated for $18,85\ Km^2$, the equation is satisfied, because its result is of $21,72\ Km^2$; superior to the $18,85\ Km^2$ – which completely satisfies the zone ZA. The following stages refer to the sensitivity analysis for the quantification or dimensioning of the number of $U_s(n_s)$, corresponding to the equations (50), (51), (52) and (53).

1) The values of the average request rates for assistance in Δt_1 (day) and Δt_2 (night) are:

$$\overline{\lambda_1} = \frac{3\ x\ 0,8066\ x\ 48,51\%\ x\ \dfrac{0,0006092953}{60\ x\ 24}\ x\ 91.047\ x\ 0,99456}{200} = 0,0224 \quad \frac{calls}{min}.$$

$$\overline{\lambda_2} = \frac{3\ x\ 0,1934\ x\ 48,51\%\ x\ \dfrac{0,0006092953}{60\ x\ 24}\ x\ 91.047\ x\ 0,99456}{100} = 0,0108 \quad \frac{calls}{min}.$$

2) Consider that the $RPA1$ is divided in a single assistance zone ZA, that is $N = 1$. Therefore, the values referring to the metrical D are given by the equations below:

$$E(D) = \sqrt{\frac{A}{4N}}\sqrt{\frac{\pi}{2}} = 2,713\ Km \ \ and \ \ V(D) = \left(0,108\frac{A}{N}\right) = 2,036\ Km^2.$$

3) The travel time TV considered constant in every way is calculated in function of $E(D)$, for a constant speed of $v = 35\ Km/h$:

$$E(TV) = 60\frac{E(D)}{v} = 4,65\ mins \ \ and \ \ V(TV) = 60^2\frac{V(D)}{v^2} = 5,98\ mins^2.$$

4) After determining the travel time and the estimation for the times TP, TA, and TD, we find the expected value of the service time $E(TS) = 17,30\ mins$ and $V(TS) = 31,91\ mins^2$.

5) The calculation of the dispersion coefficient $c = 0,326$ is used for the determination of the interpolation of $\overline{W}$. 6) After the calculation of the service time $E(TS)$ it is necessary to calculate the average assistance rate $\overline{\mu}$. Thus with the probability $P(n_s, a)$, n_s, $\overline{\lambda_1}$, $\overline{\lambda_2}$ and $\overline{\mu}$ we determine the values of $\overline{W_M}$, $\overline{W_D}$ and, afterwards $\overline{W}$:

$$\overline{\mu} = \frac{1}{E(TS)} = 0,0578\frac{calls}{min}.$$

7) For the dimensioning of the number of service units (n_s), we need to apply the Equation (50) ($P(n_s, a)$), Equation (51) ($\overline{W_M}$), Equation (52) ($\overline{W_D}$), for $n_s = 1$ and Equation (53) ($\overline{W_D}$), para $n_s > 1$. The values of $a_1 = 0,387$ and $a_2 = 0,187$, that are used as entry in Erlang's table, for the calculation of $P(n_s, a)$, with $n_s = 1, 2\ldots$ until the convergence of $\overline{W} \approx 0$ (SHAPIRO, 2001) and (KMENTA, 1990). Then for the day and night periods the minimum departure values for n_s is a U_s. Observe the calculations below:

1) Day period

For $n_s = 1$,

$P(1,\ 0,387) = 0,387$, $\overline{W_M} = 10,93\ min$, $\overline{W_D} = 5,466\ min$ and $\overline{W} = 6,047\ min$.

For $n_s = 2$,

$P(2,\ 0,387) = 0,0627$, $\overline{W_M} = 0,6727\ min$, $\overline{W_D} = 0,4625\ min$ and $\overline{W} = 0,4848\ min$.

For $n_s = 3$,

$P(3,\ 0,387) = 7,1652 \times 10^{-3}$, $\overline{W_M} = 0,0475\ min$, $\overline{W_D} = 0,0356\ min$ and $\overline{W} =\approx 0$.

2) Night period

For $n_s = 1$,

$P(1,\ 0,187) = 0,187$, $\overline{W_M} = 3,98\ min$, $\overline{W_D} = 1,99\ min$ and $\overline{W} = 2,20\ min$.

For $n_s = 2$,

$P(2,\ 0,187) = 0,0158$, $\overline{W_M} = 0,151\ min$, $\overline{W_D} = 0,101\ min$ and $\overline{W} = 0,106\ min$.

For $n_s = 3$,

$P(3,\ 0,187) = 9,5 \times 10^{-4}$, $\overline{W_M} = 5,4 \times 10^{-3}\ min$, $\overline{W_D} = 7,16 \times 10^{-4}\ min$ and $\overline{W} \approx 0$.

Conclusions

a) The effective need of service units for the day period is of $n_s = 3$. But, with $n_s = 2$ – reduction of 33,33% it is necessary to neglect the travel time

$(E(TV) = 4,65 \text{ min})$ in a percentage of $\left(\dfrac{29,08 \ sec}{60 \times 4,65 \ sec}\right) = 10,42\%$. This negligence causes a risk to the injured in a way, for not assisting him/her in the critical time – time to save the life;

b) In the night period the need is also of $n_s = 3$. In case of choosing 2 service units, the travel time must be neglected at about 2,27%, but with more ease because of a possible increase in the commercial speed of the U_s. Thus, adopting the variance interval as the average wait time in queue $\overline{W} = 29,08$ seconds would augment the $E(TV)$ to about 10,42%. So for a user population of the hospital HR with 91.047 users, 3 service units are satisfactory for the day period and 3 service units are satisfactory for the night period considering $\overline{W} \approx 0$;

c) Observe that in the day period, considering two $(n_s = 2)$ or three $(n_s = 3)$ service units, the probability of a request in the service station of the ZA, that contemplates the HR, of not finding a U_s is of $0,0627$ and of $0,000716$, respectively. This way, the increase of 50% in the service units provokes a reduction of 99% in this probability, since it is little representative. It is emphasized that the usage factor, that measures the effectiveness of the fixed costs of the U_s, represents an efficiency indicator in relation to the costs. With the increase of this n_s this usage factor decreases at 33,33%. As the fixed costs represents about 75% of the total cost, this consideration $(n_s = 3)$ imputes in an increase of the total cost for the U_s at about 25%;

d) For the night period there are 3 $(n_s = 3)$ service units. The probability of a request in the service station of the ZA, that contemplates the HR, of not finding a U_s is of $0,0158$ $(n_s = 2)$ and $0,00095$ $(n_s = 3)$. All the probabilities are very small; configuring as excellent results. It is highlighted that for the efficiency indicator of the system, the choice of $n_s = 3$ is much more favorable;

e) The $RPA1$ has a population of 77.607 inhabitants. Thus, the ZA that contemplates the user population of HR has 91.047 users. This way, the 13.440 remaining users can be considered in an extra area of $\dfrac{13.440 \ hab}{4832 \ hab} \ Km^2 = 2,89 \ Km^2$, where a satellite station with one or two service units would be allocated, probably in an area with high chances of accidents, as for example the "Conde da Boa Vista" avenue where the occurrence of people getting run over by cars has increased in a significant way. Another option is the "Complexo Salgadinho" in the division between the cities of Recife and Olinda. All in all, this user demand of 13.440 much likely come from other assistance zones;

f) In case of the user population being smaller than the population of the ZA it is necessary to relocate this assistance deficit to another close zone and recalculate its coverage area, as well as the increase of beds directed at the hospital where the assistance units station is located. Then, it is necessary to recalculate the increase of U_s units that must supply these new attendances;

g) When estimating the speed $v = 20Km/h$, the results of the U_s's dimensioning differ very little. For the day period, the effective necessity of service units remains in $n_s = 3$. But, for $n_s = 2$ and due to the substantial increase in the $E(TV)$, the risk in not assisting the injured individual in critical time due to traffic jams is $\frac{35,88\ seg}{488,4\ seg} = 7,36\%$. This number in relation to the previous result $(10,42\%)$ is about 30% smaller. The impedance of adopting $n_s = 2$ is based on the high probability $(0,0685)$ of lacking U_s, when the requests are made.

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- *RESUMO: Freidenfelds (1980) introduziu para a modelagem da demanda em diversos sistemas de transporte, os conceitos de teoria das filas e estudou o problema de expansão da capacidade do sistema de transporte como um processo aleatório de nascimento e morte, mostrando que é possível se adaptar o modelo estocástico de crescimento da demanda para um modelo determinístico. Souza (1996) aplicou esta teoria para predizer a expansão dos sistemas de atendimento emergencial. A modelagem integrada aos estabelecimentos hospitalares – atendimento emergencial e remoções inter-hospitalares, apesar de ser um serviço de mercado estritamente restrito, à medida que novas soluções vão sendo desenvolvidas, novos conhecimentos vão sendo agregados a um custo cada vez menor (GOLDBERG, 2004). As Centrais de Regulação que contém os Postos ou Estações de serviço representam o elemento ordenador e orientador dos Sistemas Estaduais de Urgência e Emergência. Essas Centrais devem ser estruturadas em todos os níveis, organizando a relação entre os vários serviços, qualificando o fluxo dos pacientes no Sistema e gerando uma porta de integração aos estabelecimentos hospitalares, por meio dos quais os pedidos de socorro são recebidos, avaliados e hierarquizados. Estas regras devem ser seguidas por todos os serviços, sejam públicos ou privados. Pode-se citar, a título de exemplo, que para os serviços emergenciais uma medida bastante usada é a maximização da utilização da U_s ou a minimização do tempo resposta (TR), entre qualquer usuário do sistema de transporte e o estabelecimento hospitalar mais próximo.*

- *PALAVRAS-CHAVE: População usuária; tempo de viagem; tempo resposta; tempo máximo resposta; unidade de serviço; teoria das filas; processo de Poisson.*

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