EUCALYPTUS GRANDIS X EUCALYPTUS UROPHYLLA GROWTH CURVE IN DIFFERENT SITE CLASSIFICATIONS, CONSIDERING RESIDUAL AUTOCORRELATION

Walleff da Silva e SILVA[1]
Felipe Augusto FERNANDES[1]
Fabiana Rezende MUNIZ[2]
Joel Augusto MUNIZ[1]
Tales Jesus FERNANDES[1]

▪ ABSTRACT: Brazil is a major producer in the timber sector, mainly with the use of wood from species of the genus Eucalyptus, with 26.1% of planted forests located in Minas Gerais. Researchers and manufacturers have been searching for techniques with the objective of making full use of these forests, with a primary focus on greater growth. A modeling of growth curves is an alternative for the estimation of floral production and an important aid tool for the researcher's decision making. Growth curves are commonly studied by nonlinear regression models, which have important assumptions that if not met should be added to the model. The present work aims to select among nonlinear Logistic, Gompertz and von Bertalanffy regression models the most suitable to describe the growth in wood volume of *Eucalyptus urophylla x Eucalyptus grandis* hybrids in three Forest Site categories, including whether assumption deviations are required. Methods were executed by the Gauss-Newton iterative method implemented in *nls()* and it *gnls()* functions of the R software. Determination coefficient, Akaike information criterion (AICc) and Residual Standard Deviation (RSD) were used as selection evaluators of the best model. The results demonstrate that for all site categories, the Gompertz model with addition of autoregressive parameters AR (1) is the most appropriate to describe the growth in wood volume of *Eucalyptus urophylla x Eucalyptus grandis* hybrids. The addition of the first-order autoregressive parameter does not affect the quality of fit, but it is the correct procedure. Site I, which presents the largest trees according to pre-defined variations, recorded 308 m^3/ha of wood volume, followed by 286 m^3/ha and 263 m^3/ha for Sites II and III, respectively. The time for Site III to reach the maximum point of volume growth is between the fourth and sixth year, while the other sites are more precocious, reaching this point between the second and third year.

[1]Universidade Federal de Lavras - UFLA, Instituto de Ciências Exatas e Tecnológicas, Departamento de Estatística, CEP: 37200-900, Lavras, MG, Brasil. E-mail: *walleffsilva@gmail.com; fernandesfelipest@gmail.com; joamuniz@ufla.br; tales.jfernandes@ufla.br*
[2]Escola Superior de Agricultura "Luiz de Queiroz" - Esalq-USP, CEP: 13418-900, Piracicaba, SP, Brasil. E-mail: *fabiana.muniz@hotmail.com.br*

▪ KEYWORDS: *Eucalyptus*, growth curves, forest site categories, nonlinear models.

1 Introduction

Planted wood of the genus *Eucalyptus* has great potential due to its availability in a short period of time and use in the production of raw material for cellulose, charcoal and poles. According to data from IBGE (2018), Brazil has around 9.9 million hectares of planted forests, of which 7.5 million are eucalyptus, with the southeastern region being responsible for 42.3% of the total planted area in Brazil, mainly in the states of Minas Gerais (26.1%) and São Paulo (12.2%). For maximum use of these hectares, researchers have been looking for more efficient ways to use the forest, mainly in the breeding of trees and forest management.

The growth of a tree consists of the thickening and elongation of roots, trunk and branches and increase in the number of leaves, resulting in changes in tree volume and shape (SCOLFORO, 2006). In a forestry company, production planning is essential and, in this context, forest growth and production modeling is a tool that assists researchers in making decisions. The growth and production of a forest stand depends on age, productive capacity or site, degree of utilization of productive potential and silvicultural treatments. Other strategies to assist growth are better management of fertilizers, the use of high-quality seedlings and the control of weeds, pests and diseases (ELLI *et al.*, 2019).

Among tree characteristics, height and diameter constitute an important variable, which is determined or estimated essentially for the calculation of volume and for the calculation of height and volume increments. Retslaff *et al.* (2015) point out that, in forest inventories, determining the height of all trees can lead to more errors than estimating it, due to visualization difficulties in addition higher demand of time and costs. Mendonça *et al.* (2017) reported that the wood volume, both present and future, is the most important item in forest planning.

For Scolforo (2006), the use of data in which height and age are related is the most practical, efficient and consistent method to use data in the classification of forest sites, because height correlates very well with the development potential of most species used in reforestation. Therefore, the greater the height, the better the site quality. According to Bila (2012), the practical importance of the classification of sites in terms of productivity is related to the decrease in costs for forestry companies with permanent installments, with the provision of information that will define management techniques (precision forestry) more adapted to a given situation, considering sustainable aspects and economic profitability.

According to Coutinho *et al.* (2017), when grouping all trees and analyzing their increments, it was observed that the production curve has a sigmoidal shape, in which the first phase corresponds to the juvenile age, the second to the mature age and the third to the senile age. Thus, the optimum cutting age, from the technical point of view, is when the population cycle allows obtaining the largest wood volume per unit of area per year. This age varies according to the growth curve of the forest and must be evaluated for each plantation using inventory techniques and forest biometry.

The expression 'growth curve' usually refers to sigmoidal curves that represent the behavior of dimension measurements over time. Animal, plant and fruit growth curves are commonly studied using nonlinear regression models, as in Fernandes *et al.* (2019), Jane *et al.* (2019) and Silva *et al.* (2020), respectively. Specifically in the forestry area, some works in literature have been using nonlinear models to describe the growth curve. Venduscrolo *et al.* (2016) evaluated modeling using nonlinear regression and artificial neural networks to estimate the height of eucalyptus trees. Coutinho *et al.* (2017) adjusted nonlinear regression Weibull, Logistic, Gompertz, Schumacher and Chapman-Richards models to assess the growth pattern and describe the probabilistic distribution of the annual increase in the diameter of *Cryptomeria japonica*. Santos *et al.* (2017) using nonlinear models, estimated characteristics such as dominant height, average diameter, basal area and volume with bark according to age.

According to Mendonça *et al.* (2017), when using regression models, important assumptions are added to the problem of establishing a relationship among variables of interest. Prado *et al.* (2020) emphasize that the assumption of residual independence is almost always not accepted when studying growth curves, as measures are taken repeatedly in the same individual, the residue of an observation can be associated with the residue of adjacent observations, that is, residues are autocorrelated. Studies on growth in volume of forests have not taken this assumption deviation into account.

Thus, the aim of the present work was to select among the nonlinear regression models Logistic, Gompertz and von Bertalanffy, the most adequate to describe the growth in wood volume of *Eucalyptus urophylla x Eucalyptus grandis* hybrids in three site classifications, considering assumption deviations if necessary.

2 Material and Methods

2.1 Material

Data from Gonçalves *et al.* (2017) referring to the planting of *Eucalyptus grandis x Eucalyptus urophylla* hybrids that were historic and active since the second cycle, with reference in December 2012, were used. For the estimation of the growth curves of eucalyptus plantations information on age (years) and volumetric production of wood (m^3/ha) from monoclonal plantations were used.

Data from were obtained from the Forest Inventory of Fibria Celulose S.A., Aracruz branch, located at Aracruz, Espírito Santo, Brazil. The average temperature of the region is 28°C and the average annual rainfall is 1,200 mm. The predominant soil type in the region is Distrophic Red-Yellow Latosol and Red-Yellow Podzol (GONÇALVES *et al.*, 2017).

The forest area was divided into 3 site classes, with large trees (site I), medium trees (site II) and small trees (site III). According to Gonçalves *et al.* (2017), site classification included three steps: adjustment of a general model, projection of the dominant height and site classification defining the upper and lower limits of each class, creating the 3 site classes. The wood volume (in m^3/ha) from the first to the 15^{th} year was then calculated.

2.2 Methods

The growth analysis of *Eucalyptus grandis x Eucalyptus urophylla* plants was performed using nonlinear models. Logistic (1), Gompertz (2) and von Bertalanffy (3) models were adjusted.

$$Y_i = \frac{\alpha}{1 + e^{k(\beta - x_i)}} + \epsilon_i \tag{1}$$

$$Y_i = \alpha e^{-e^{k(\beta - x_i)}} + \epsilon_i \tag{2}$$

$$Y_i = \alpha \left[1 - \frac{e^{k(\beta - x_i)}}{3}\right]^3 + \epsilon_i \tag{3}$$

where Y_i represents the volume of trees in the site in m^3 in the *i-th* observation; α represents the maximum horizontal asymptote, that is, the maximum volume that a tree of that site can reach; k represents the growth rate (the higher the k, the less time the tree takes to reach α); β is interpreted as the abscissa of the inflection point, from which growth decelerates, x_i is the *i-th* year of measurement. ϵ_i corresponds to the random error, which is assumed and is independent and identically distributed following normal distribution with mean of zero and constant variance, that is $\epsilon_i \sim N(0; \sigma^2)$).

For the analysis of residues, Shapiro-Wilk tests (SHAPIRO and WILK, 1965) were used to verify normality of residues, in which the null hypothesis is that the residues follow a normal distribution, the Breuch-Pagan test (BREUSCH and PAGAN, 1979) to assess the homogeneity of the variance, whose null hypothesis is that the residues are homoscedastic and the Durbin-Watson test (DURBIN and WATSON, 1950) to check for the existence of residual autocorrelation, the null hypothesis for this test is the independence of waste. If there is violation of the independence assumption, it will be added one parameter autoregressive first-order AR (1) to the model.

Heterogeneity of variances and residual autocorrelation are characteristics inherent to growth data over time that, in most works, are not considered in the modeling, which can lead to inaccurate estimates and results (MAZZINI et al., 2005; MENDES et al ., 2008; PRADO, SAVIAN and MUNIZ, 2013).

When residual dependence was identified, in order to obtain reliable parameter estimates, it was necessary to incorporate this feature in the modeling. Thus the error vector was as described below:

$$\epsilon_i = \phi_1 \epsilon_{i-1} + ... + \phi_p \epsilon_{i-p} + \upsilon_i$$

where: i = 2, 3, ..., 15 is the number of years in which the volume was estimated; ϵ_i is the residue estimated in the i-th year; ϕ_1 is first-order autoregressive parameter; ϵ_{i-1} is the residual estimated for the year immediately preceding the i-th year; and ϕ_p is the autoregressive parameter of order p; ϵ_{i-p} is the residual estimated for p years before the i-th year and υ_i is the random error of the model that follows normal distribution with mean zero and constant variance. When residues are independent, then parameters $\phi_1, ..., \phi_p$ are null, hence $\epsilon_i = \upsilon_i$ (SILVA *et al.*, 2020).

In order to compare and evaluate model adjustments, the corrected Akaike information criterion (AICc) was used, and the model with the lowest AICc estimation was chosen. Regarding the determination coefficient (R^2), the best model is the one with the highest value. In addition, the Residual Standard Deviation (RSD) was used to compare adjustments, with those with the lowest RSD values being considered the best models. To illustrate the estimates of adjusted models, graph analysis was used.

The estimates of parameters and figures presented here were obtained by the iterative Gauss-Newton method implemented in *nls ()* and *gnls ()* functions of the R software. The significance of parameters was verified using the t test at 5% significance level.

3 Results and Discussion

Initially, the above models were adjusted and analysis of residues was performed (Table 1). The assumptions that must be assumed by the error (normality, independence and homogeneity of variances) were evaluated, respectively, by the Shapiro-Wilk (SW), Durbin-Watson (DW) and Breusch-Pagan (BP) tests, considering 5% significance level.

Table 1 - p-values of Shapiro-Wilk (SW), Durbin-Watson (DW) and Breusch-Pagan (BP) tests used to validate the assumptions of the analysis of residues for Logistics, Gompertz and von Bertalanffy models for wood volume in the three sites

Site	Test	Logístico	Gompertz	von Bertalanffy
Site I	SW	0.3567	0.1560	0.4013
	DW	**<0.001**	**<0.001**	**<0.001**
	BP	0.2189	0.8900	0.5038
Site II	SW	0.4863	0.2776	0.3602
	DW	**<0.001**	**<0.001**	**<0.001**
	BP	0.2308	0.8900	0.0523
Site III	SW	0.7398	0.5211	0.8312
	DW	**<0.001**	**0.0356**	**<0.001**
	BP	0.0722	0.2257	0.8355

It was observed that the assumptions of normality and homogeneities of variance for the random error were validated. However, for all models and for all site classifications, the Durbin-Watson test was significant, violating the assumption of independence, so it was necessary to include a first-order autoregressive parameter AR (1) in the model.

In a study on the growth of pepper cultivar 'Doce', Jane *et al.* (2019) observed that there was no need for the inclusion of the autoregressive parameter. However, Muniz *et al.* (2017) added this parameter to their model and found significant improvement in the growth adjustment of cocoa fruits, including significant reduction in the residual standard deviation, making estimates more reliable.

Pereira *et al.* (2016), considering the residual autocorrelation with the addition of the first-order autoregressive parameter AR (1), obtained the lowest values in all densities and irrigation regimes analyzed for the height growth of coffee plants. Describing the growth of coconut fruits, Prado *et al.* (2013) showed that adjustment of the logistic model to experimental data of longitudinal external diameter (LED), considering the first-order autoregressive structure for residues, is adequate in the description of data and resulted in estimates of parameters quite consistent with those reported in literature.

Prado *et al.* (2020) reported that in the description of coconut growth, both for longitudinal and cross-sectional diameter of the internal cavity, it was necessary to incorporate the residual dependence modeling, since this assumption was not met. Studying the blackberry growth curve, Silva *et al.* (2020) pointed out the need to add parameter ϕ_1 for some cultivars and not for others, thus reinforcing the need to always conduct the analysis of residues, even if for a specific cultivar or variable, it was not necessary to use the autoregressive parameter.

The parameter estimates, obtained by the Gauss-Newton iterative method, for the wood volume of Site I and II are contained in Tables 2 and 3, observing the selection criteria of the models (higher R^2, lower AICc and SRD values) and comparing the choices of adequate adjustments according to the addition or not of the autoregressive parameter. It could be observed that there is a change in the indication of the model, in these scenarios, without the addition of the first-order autoregressive parameter AR (1). Observing the previously mentioned selection criteria, von Bertalanffy model proved to be satisfactory, while with the inclusion of this parameter, the adjustment quality evaluators of Gompertz model presented better results compared to the others.

For Sites I and II, analyzing models adjusted without adding the autoregressive parameter and observing the evaluators to select the best adjustment with lower $AICc$, high R^2 and lower RSD, these criteria indicated the von Bertalanffy model as the most appropriate. However, with the violation of the assumption of independence, it was necessary to add the first-order autoregressive parameter AR (1) to the model in both sites. In this scenario, $AICc$ selected the von Bertalanffy model, while R^2 and RSD selected the Gompertz model as the best fit.

Table 2 shows that the estimates for the maximum wood volume without adding the autoregressive parameter ranges from 303.2850 m^3/ha to 313.3403

Table 2 - Estimates and standard error (SE) of parameters and adjustment quality evaluators (R^2, AICc, SRD) for parameters of Logistic, Gompertz and von Bertalanffy models for the description of *Eucalyptus grandis x Eucalyptus urophylla* wood volume for site I

Model	Parameters	Estimates	Standard error	R^2	AICc	DRP
Logístico sem AR (1)	α	303.2850	4.3027	0.9883	119.5174	10.3517
	β	3.9618	0.1221			
	k	0.6521	0.0488			
Logístico com AR (1)	α	335.5719	23.4811	0.7561	99.8195	40.3964
	β	3.2009	0.1988			
	k	0.6258	0.0517			
	φ	0.9942	-			
Gompertz sem AR (1)	α	309.5392	2.4282	0.9977	97.5329	4.9746
	β	3.0673	0.0551			
	k	0.4473	0.0163			
Gompertz com AR (1)	α	308.6503	6.1389	**0.9959**	86.2107	**6.5830**
	β	2.9883	0.0985			
	k	0.4788	0.0209			
	φ	0.8878	-			
von Bertalanffy sem AR (1)	α	313.3403	1.8960	**0.9987**	**87.2063**	**3.5283**
	β	2.6023	0.0038			
	k	0.3799	0.0099			
von Bertalanffy com AR (1)	α	289.8084	8.3144	0.9641	**80.5235**	24.5661
	β	2.8119	0.0611			
	k	0.4359	0.0169			
	φ	0.9958	-			

m^3/ha, while for the model with addition of the autoregressive parameter, the maximum wood volume ranges from 289.8084 m^3/ha to 335.5719 m^3/ha. According to Pereira *et al.* (2016), the moment when plants reach maximum growth rate, shifting from a period of accelerated growth to the inhibition period (inflection point), is represented by parameter β. Thus, it was observed that the deceleration in volume growth in Site I is between the second and fourth years.

For Site II, according to Table 3, it was found that the maximum estimated wood volume ranges from 277.8772 m^3/ha to 291.0234 m^3/ha for models without the addition of the autoregressive parameter. For models without adding the autoregressive parameter, there is a slowdown in the wood volume growth between the third and fourth year. Observing models with the addition of the autoregressive parameter, variation from 272.4648 m^3/ha to 314.3186m^3/ha was observed for maximum wood volume for Site II.

Table 3 - Estimates and standard error (SE) of parameters and adjustment quality evaluators (R^2, AICc, SRD) for parameters of Logistic, Gompertz and von Bertalanffy models for the description of *Eucalyptus grandis x Eucalyptus urophylla* wood volume for Site II

Model	Parameters	Estimates	Standard error	R^2	AICc	DRP
	α	277.8772	3.5632			
Logístico sem AR (1)	β	4.5608	0.1099	0.9936	107.9945	7.5820
	k	0.5748	0.0335			
	α	314.3186	18.0746			
Logístico com AR (1)	β	3.8371	0.1930	0.7803	87.0017	38.7508
	k	0.5102	0.0374			
	φ	0.9971	-			
	α	285.8579	1.4680			
Gompertz sem AR (1)	β	3.5540	0.0338	**0.9993**	74.7038	2.4995
	k	0.3853	0.0078			
	α	286.0150	2.8097			
Gompertz com AR (1)	β	3.5465	0.0665	**0.9995**	65.8027	**2.8747**
	k	0.3940	0.0107			
	φ	0.8123	-			
	α	291.0234	1.5460			
von Bertalanffy sem AR (1)	β	3.0191	0.0298	**0.9995**	**71.8332**	**2.2714**
	k	0.3220	0.0062			
	α	272.4648	4.2537			
von Bertalanffy com AR (1)	β	3.3027	0.0399	0.9718	**56.0140**	18.6919
	k	0.3521	0.0072			
	φ	0.8123	-			

For Site III, as shown in Table 4, a different behavior from the other Sites was observed. The selection criteria agree to select the Gompertz model as the best fit, both with the addition of the AR parameter (1) and without the addition. The estimates for the chosen models are close to each other with small variations. For models without the addition of the autoregressive parameter, it was observed that the maximum wood volume growth is between 245.9575 m^3/ha and 277.3292 m^3/ha. However, with the addition of the autoregressive parameter, the variation is between 263.8590 m^3/ha and 270.9575 m^3/ha. These values are closer to the estimated maximum volume of 218.6599 m^3/ha for *Eucalyptus camaldulensis x Eucalyptus urophylla* obtained by Mendonça *et al.* (2017). Analyzing the estimates for the inflection point (parameter β), which indicates the point of maximum wood volume growth, this point occurs later than in the other Sites, between the fourth and sixth year.

Thus, it could be concluded that the Gompertz model with the addition of the

Table 4 - Estimates and standard error (SE) of parameters and adjustment quality evaluators (R^2, AICc, SRD) for parameters of Logistic, Gompertz and von Bertalanffy models for the description of Eucalyptus grandis x Eucalyptus urophylla wood volume for site III

Model	Parameters	Estimates	Standard error	R^2	AICc	DRP
Logístico sem AR (1)	α	245.9575	3.2838	0.9972	94.2338	4.4572
	β	6.1357	0.1110			
	k	0.4307	0.0173			
Logístico com AR (1)	α	270.9575	9.0340	0.9117	70.6548	9.9879
	β	5.6360	0.1737			
	k	0.3718	0.0179			
	φ	0.9856	-			
Gompertz sem AR (1)	α	263.7795	0.4835	**0.9999**	**23.3763**	**0.4200**
	β	4.9471	0.0108			
	k	0.2638	0.0020			
Gompertz com AR (1)	α	263.8590	0.5660	**0.9999**	**24.6178**	**0.4200**
	β	4.9487	0.0130			
	k	0.2637	0.0014			
	φ	0.2303	-			
von Bertalanffy sem AR (1)	α	277.3292	3.2450	0.9851	72.4399	2.1552
	β	4.2707	0.0544			
	k	0.2065	0.0054			
von Bertalanffy com AR (1)	α	264.7317	6.0754	0.9851	57.3795	9.8254
	β	4.5445	0.0908			
	k	0.2175	0.0069			
	φ	0.9943	-			

autoregressive parameter is the most appropriate to describe the growth in volume of *Eucalyptus grandis x Eucalyptus urophylla* hibrids. Mendonça *et al.* (2017) did not detect residual dependence and without making comparison with other models, concluded that the Logistic model with addition of covariates is compatible with the growth in volume of *Eucalyptus camaldulensis x Eucalyptus urophylla*.

Pereira *et al.* (2014) showed that for variable plant height, the Gompertz model, sometimes with structure of independent errors, sometimes with autocorrelated errors, presented, in six situations analyzed, values closest to the one for the adjusted determination coefficient and also the smallest Akaike information criterion values, corroborating to the fact that the Gompertz model was the one that best described the height growth of coffee plants. Studying the growth curve of coffee plants, Fernandes *et al.* (2014) also concluded that the Gompertz model with incorporation of assumption deviations on the residues vector was the one that provided the best adjustments.

Pereira *et al.* (2016) reported that the Gompertz model, considering heterogeneity of variances and residual autocorrelation, presented the lowest values in all analyzed densities and irrigation regimes, indicating that this model is the one that best describes the height growth of coffee plants over time. According to Muniz *et al.* (2017), the incorporation of the first-order autoregressive parameter improved the quality of fit, with reduction of about 40% in the residual standard deviation for the Gompertz model. Venduscrolo *et al.* (2017) showed that the estimates presented values statistically equal to those observed by the t-test, indicating that the Gompertz model is efficient for estimating the dependent variable (total height) as a function of the independent variable (diameter at chest height).

Lundgren *et al.* (2017) evaluated the influence of the sampling type in the estimation of the Eucalyptus wood volume and found that the total wood volume that was measured at 7.5 years provided the value of 166.14 cubic meters for an area of 2.4 ha, result similar to the estimates for Site III. For Marangon *et al.* (2017), in the dynamics of the diametric distribution and eucalyptus production in different ages and spaces, at the age of 7 years the Site reaches its maximum accumulated volumetric productivity, and at the age of 13 years the Site reaches its minimum for the three ages analyzed, possibly due to the fact that trees of this age have higher individual volumes and smaller diametric distribution.

Regarding wood volume production for all sites, verifying estimates obtained by parameter α, it was observed that Site I can be considered the most productive and, according to Gonçalves *et al.* (2017), Site III shows slower growth and smaller production than the other sites, since its inflection point is estimated to occur later.

Coutinho *et al.* (2017) explained that the growth of trees is not regular throughout their life cycles, and for cryptomeria individuals studied, a decrease in diameter at chest height was observed over the years and that the largest increments are concentrated in the juvenile phase of trees, where naturally there is greater vigor, when compared to maturity and senescence phases. Lima *et al.* (2017) observed maturity or growth rate estimates (k) and found the occurrence of small values, mainly when the von Bertalanffy model was adjusted, and these small values indicate that plants take longer to reach maturity.

Marangon *et al.* (2017) reported that the growth and productivity level of sites may vary according to the water availability, spacing between plants, soil richness, genetic potential of plants, site capacity, etc. Almeida *et al.* (2017) analyzed the estimated wood volume for Eucalyptus sp. with high spatial resolution satellite images, observing the generated wood volume maps, and showed that it was possible to verify how much development can be uneven within the same species.

Figures 1, 2 and 3 show the graphic adjustment for the description of the *Eucalyptus grandis x Eucalyptus urophylla* wood volume for Sites I, II and III (respectively), showing the difference in prediction for models with or without the addition of the first-order autoregressive parameter AR (1).

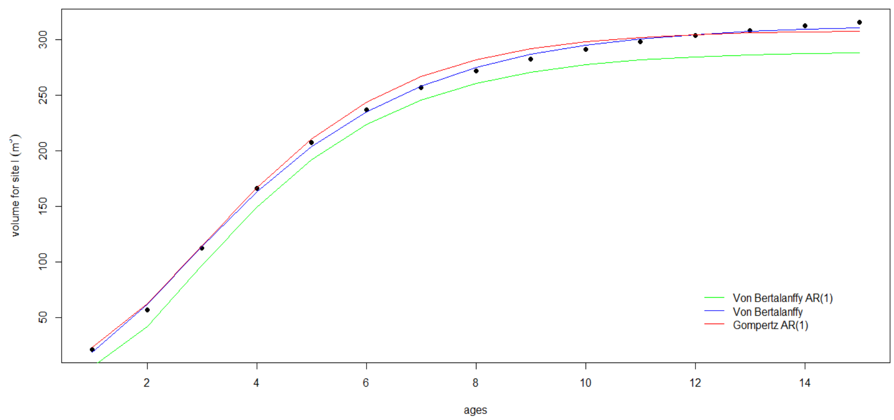

Figure 1 - Adjustment of models selected by the adjustment selection criteria in the description of the *Eucalyptus grandis x Eucalyptus urophylla* wood volume for site I.

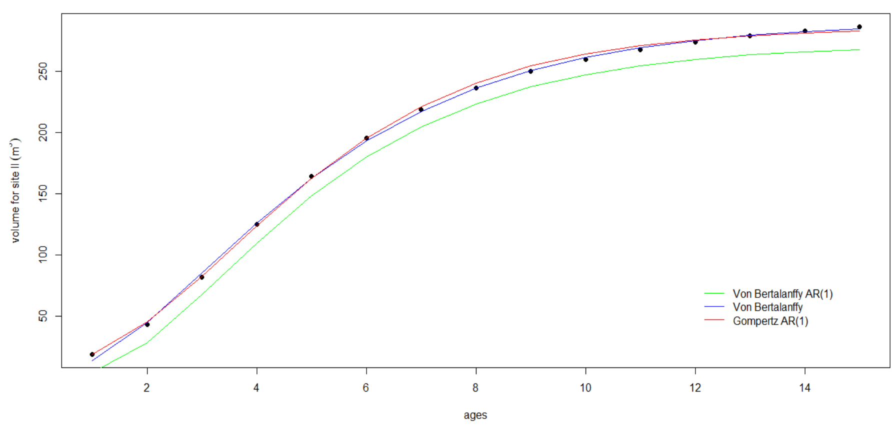

Figure 2 - Adjustment of models selected by the adjustment selection criteria in the description of the *Eucalyptus grandis x Eucalyptus urophylla* wood volume for site II.

According to results shown in Tables 2 and 3 for Sites I and II, it was observed that, in principle, the von Bertalanffy model was the preferred candidate among the other models based on the adjustment quality evaluators. However, when adding the assumption of independence, this model presented poor estimates, underestimating the asymptote (Figures 1 and 2). This evidences that the analysis of results, frequently ignored in literature, constitutes one of the most important stages when studying regression models.

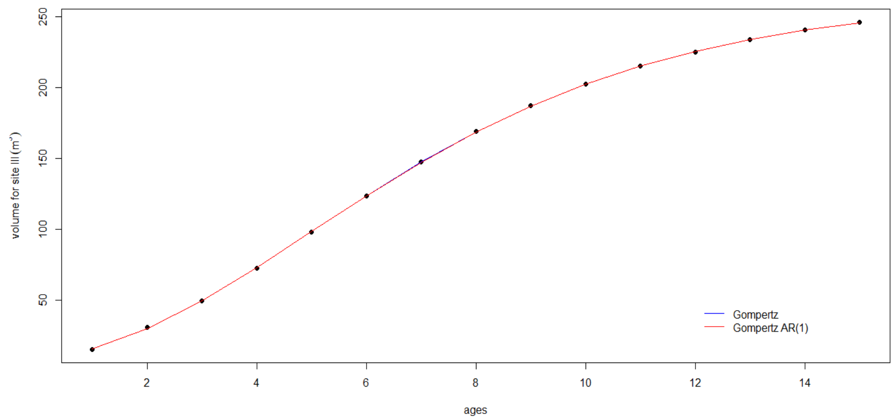

Figure 3 - Adjustment of models selected by the adjustment selection criteria in the description of the *Eucalyptus grandis x Eucalyptus urophylla* wood volume for site III.

Underestimation and overestimation in asymptote estimates were observed in other studies. Santos *et al.* (2017) found underestimation and overestimation points in the estimation of Tauari volume in the Tapajós National Forest, with tendency of overestimation for volumes of up to 5 m^3 and underestimation for volumes from 10 m^3. Gomes *et al.* (2018) observed tendency of overestimation in commercial height in the lower stratum and underestimation in the upper stratum for volumetric estimates of wood volume in the Tapajós National Forest.

Nunes *et al.* (2017), using Hohenadl-Kren, Husch, Stoate and Schumacher-Hall models for the volumetric estimation of a dense *Eucalyptus sp.* population, found significant tendency of underestimation for smaller diameter trees, while the Brenac model showed a slight tendency of overestimation for trees with diameter at chest height (DCH) between 6 and 8 cm.

Andrade (2017) used a volumetric model for *Eucalyptus urophylla* and *Eucalyptus grandis* trees aged 5-7 years developed from the form factor equation adapted to the Gompertz bio-mathematical model, presenting smaller variation of errors around the zero axis, with less pronounced tendency of underestimation. Volumetric modeling resulted in estimates similar to other studies, like Santos *et al.* (2017) that found satisfactory results using the Gompertz model for volume, presenting mean growth values similar to those found in the present work. Vendruscolo *et al.* (2017) found that the equations obtained with the Gompertz model showed adjustment statistics equal to or slightly higher than other models for estimating the height of *Tectona grandis L.f.* trees.

Despite the violation of the assumption of independence, the adjustment of models without the addition of the autoregressive parameter AR (1) was presented in result tables to enable the comparison between estimates and possible adjustment improvement, mainly due to disagreement in the selection of the best model in Sites I and II. Tables 2, 3 and 4 showed that there was an increase in the standard error of the estimate of all parameters when considering the first-order residual dependence AR (1). This increase in the standard error of the estimate was also observed by Prado *et al.* (2020), that show that incorporating residual dependence does not necessarily improve the quality of fit, but it is the most coherent, because without adding the autoregressive parameter, it would be accepted that residues are independent, when in fact they are not.

Conclusions

The Gompertz model with the addition of the first-order autoregressive parameter AR (1) was the most adequate to describe the growth in wood volume of *Eucalyptus grandis x Eucalyptus urophylla* hybrids for the three Site classifications, obtaining satisfactory estimates for wood volume.

Estimates without the autoregressive parameter AR (1) for the von Bertalanffy model, although similar to those obtained with the addition of this parameter in the Gompertz model, have become obsolete. An increase in the standard error of the estimate was also observed when incorporating AR (1) in all models, reinforcing that the addition of the first-order autoregressive parameter does not necessarily improve the quality of fit. However, it is more coherent, since the assumption of residual independence was not met.

The estimate for maximum wood volume in Site I was 308 m^3/ha, while Sites II and III showed lower results, 286 m^3/ha and 263 m^3/ha, respectively. In addition, it was verified that the time for Site III to reach the point of maximum volume growth is between the fourth and the sixth year, while the other Sites are more precocious, reaching this point between the second and third year.

Acknowledgments

This work was carried out with financial support from the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq).

SILVA, W. S.; FERNANDES, F. A.; MUNIZ, F. R.; MUNIZ, J. A.; FERNANDES, T. J. Curva de crescimento de *Eucalyptus grandis x Eucalyptus urophylla* em diferentes classificações de sítios, considerando-se Autocorrelação residual. *Rev. Bras. Biom.,* Lavras, v.39, n.1, p.122-138, 2021.

■ *RESUMO: O Brasil é um grande produtor do setor madeireiro, principalmente com a utilização de madeira de espécies do gênero Eucalyptus, sendo 26,1% das florestas plantadas localizada em Minas Gerais. Pesquisadores e madeireiros têm buscado técnicas com objetivo de obter um aproveitamento total dessas florestas, com principal foco no maior crescimento delas. A modelagem das curvas de crescimento é uma alternativa para a estimativa da produção florestal e uma importante ferramenta de auxílio para tomada decisões do pesquisador. As curvas de crescimento são comumente estudadas por meio de modelos de regressão não linear, os quais tem importantes pressuposições que se não forem atendidas devem ser acrescentadas ao modelo. O presente trabalho tem como objetivo selecionar dentre os modelos de regressão não linear Logístico, Gompertz e von Bertalanffy o mais adequado para descrever o crescimento em volume de madeira dos híbridos Eucalyptus urophylla x Eucalyptus grandis em três classificações de Sítios florestais, considerando se necessário os desvios de pressupostos. As estimativas dos parâmetros foram obtidas pelo método iterativo de Gauss-Newton implementado na função nls( ) e gnls( ) do software R. O coeficiente de determinação, critério de Akaike corrigido e o desvio-padrão residual foram utilizados como avaliadores de seleção do melhor modelo. Os resultados demonstram que para todas as classificações de Sítio, o modelo Gompertz com adição do parâmetro autorregressivo AR (1) é o mais adequado para descrever o crescimento em volume de madeira dos híbridos Eucalyptus urophylla x Eucalyptus grandis. A adição do parâmetro autorregressivo de primeira ordem não necessariamente melhora a qualidade de ajuste, entretanto é o correto a ser feito. O Sítio I, que apresenta as maiores árvores de acordo com as classificações pré-estabelecidas, obteve assíntota de 308* m^3/ha *no volume de madeira, seguido por 286* m^3/ha *e 263* m^3/ha *dos Sítios II e III, respectivamente. O tempo para o Sítio III atingir o ponto de máximo crescimento em volume, está entre o quarto e o sexto ano, enquanto que os demais Sítios são mais precoces, atingindo este ponto entre o segundo e terceiro ano.*

■ *PALAVRAS-CHAVE: Eucalyptus; Curvas de Crescimento; Sítios Florestais.*

References

ANDRADE, V. C. L. de. Modelos Volumétricos de Dupla Entrada para Aplicar em Povoamentos Florestais Brasileiros. *Floresta Ambiente*, v.24, e00135415, 2017.

BILA, J. M.; SANQUETTA, C. B.; DO AMARAL MACHADO, S. Classificação de sítios com base em fatores edáficos para Pinus caribaea var. hondurensis na região de Prata, Minas Gerais. *Floresta*, v.42, n.3, p. 465-474, 2012.

BREUSCH, T. S.; PAGAN, A. R. A Simple test for heteroscedasticity and random coefficient variation. *Econometrica*, New York, v.47, n.5, p.1287-1294, sep. 1979.

COUTINHO, V. M.; CORTE, A. P. D.; SANQUETTA, C. R.; RODRIGUES, A. L; SANQUETTA, M. N. I. Modelagem do crescimento de Cryptomeria japonica por análise de tronco parcial. *Pesquisa Florestal Brasileira*, online, v.37, n.90, p.93-98, 2017.

DEMACEDO, F. L.; De OLIVEIRA SOUSA, A. M.; GONÇALVES, A. C.; SILVA, H. R.; RODRIGUES, R. A. F. Estimativa do volume de madeira para Eucalyptus sp. Com imagens de satélite de alta resolução espacial. *Scientia Forestalis/Forest Sciences*, v.45, n.114, p.237-247, 2017.

DURBIN, J.; WATSON G. S. Testing for serial correlation in least squares regression I. *Biometrika*, London, v.37, n.3/4, p.409-428, 1950.

ELLI, E. F.; SENTELHAS, P. C.; FREITAS, C. H.; CARNEIRO, R. L.; ALCARDE ALVARES, C. Assessing the growth gaps of Eucalyptus plantations in Brazil -Magnitudes, causes and possible mitigation strategies. *Forest Ecology and Management*, v.451, e117464, 2019.

FERNANDES, F. A.; FERNANDES, T. J.; PEREIRA, A. A.; MEIRELLES, S. L. C.; COSTA, A. C. Growth curves of meat-producing mammals by von Bertalanffy's model. *Pesquisa Agropecuária Brasileira*, v.54, e01162, 2019.

FERNANDES, T. J.; PEREIRA, A. A.; MUNIZ, J. A.; SAVIAN, T. V. Seleção de modelos não lineares para a descrição das curvas de crescimento do fruto do cafeeiro. *Coffee Science*, v.9, n.2, p.207-215, 2014.

GOMES, K. M. A. G.; RIBEIRO, R.; GAMA, J. R. V. Eficiência na estimativa volumétrica de madeira na Floresta Nacional do Tapajós. **Nativa**, v.6, n.2, p.170-176, 2018

GONCALVES, J. C.; OLIVEIRA, A. D. de; CARVALHO, S. P. C.; GOMIDE, L. R. Análise econômica da rotação florestal de povoamentos de eucalipto utilizando a simulação de Monte Carlo. *Ciência Florestal*, v.27, n.4, p.1339-1347, 2017.

IBGE. INSTITUTO BRASILEIRO DE GEOGRAFIA E ESTATÍSTICA. *Produção da Extração vegetal e Silvicultura*, Rio de Janeiro, v.33, p.1-8, 2018.

JANE, S. A.; FERNANDES, F. A.; SILVA, E. M; MUNIZ, J. A.;, FERNANDES, T. J. Comparação dos modelos polinomial e não lineares na descrição do crescimento de pimenta. *Revista Brasileirade Ciencias Agrarias*. Recife, v.14, n.4, 2019.

LIMA, K. P.; MORAIS, A. R.; VIEIRA, N. M. B.; VILLA, F.; ANDRADE, M. J. B. Nonlinear models for use in description of Boron accumulation in diferent parts of Jalo beans. *Revista Brasileira de Biometria*, Lavras, v.35, n.4, p.834-861, 2017.

LUNDGREN, W. J. C.; SILVA, J. A. A. da; FERREIRA, R. L. C. Estimação do Volume de Eucaliptos por Krigagem e Cokrigagem no Semiárido Pernambucano. *Floresta Ambient*, v.24, e00140415, 2017.

MAZZINI, A. R. de A.; MUNIZ, J. A.; SILVA, F. F. e; AQUINO, L. H. de. Curva de crescimento de novilhos Hereford: heterocedasticidade e resíduos autoregressivos. *Ciência Rural*, v.35, n.2, p.422-427, 2005.

MARANGON, G. P.; COSTA, E. A.; ZIMMERMANN, A. P. L.; SCHNEIDER, P. R.; SILVA, E. A. Dinâmica da distribuição diamétrica e produção de eucalipto em diferentes idades e espaçamentos. *Revista de Ciências Agrárias*, v.60, n.1, p.33-37, 2017.

MENDES, P. N.; MUNIZ, J. A.; SILVA, F. F. e; MAZZINI, A. R. de A. Modelo logístico difásico no estudo do crescimento de fêmeas da raça Hereford. *Ciência Rural*, Santa Maria, v.38, n.7, p.1984-1990, 2008.

MENDONÇA, A. R.; CALEGARIO, N.; SILVA, G. F.; CARVALHO, S. P. C. Growth and yield models for eucalyptus stands obtained by differential equations. *Scientia Agricola*, v.74, n.5, p.264-370, 2017.

MUNIZ, J. A.; NASCIMENTO, M. S.; FERNANDES, T. J. Nonlinear Models For Description Of Cacao Fruit Growth With Assumption Violations. *Rev. Caatinga*, Mossoró , v.30, n.1, p.250-257, 2017.

NUNES, J. S.; SOARES, T. S. Estimativas volumétricas para um povoamento adensado de Eucalyptus sp. em regime de curta rotação. *Revista de Agricultura Neotropical*, Cassilândia-MS, v.4, n.4, p.77-86, 2017.

PEREIRA, A. A.; MORAIS, A. R. de; SCALCO, M. S.; FERNANDES, T. J. Descrição do crescimento vegetativo do cafeeiro cultivar Rubi MG 1192 utilizando modelos de regressão. *Coffee Science*, v.9, p.266-274, 2014.

PEREIRA, A, A.; MORAIS, A. R.; SCALCO, M. S.; FERNANDES, T. J. Modelagem não linear do crescimento em altura do cafeeiro irrigado e não irrigado em diferentes densidades. *IRRIGA*, v.1, n.1, p.140-149, 2016.

PRADO, T. K. L; MUNIZ, J. A.; SAVIAN, T. V.; SÁFADI, T. Ajuste do modelo Logístico na descrição do crescimento de frutos de coqueiros não verde por meio de algoritmos iterativos MCMC. *Revista Brasileira de Biometria*, v.31, n.2, p.216-232, 2013.

PRADO, T. K. L.; SAVIAN, T. V.; FERNANDES, T. J.; MUNIZ, J. A. Study of the growth curve of the internal cavity of fruit dwarf green coconut. *Revista Ciência Agronômica*, v.51, n.3, e20154591, 2020.

R DEVELOPMENT CORE TEAM. R: a language and environment for statistical computing. Vienna: R Foundation for Statistical Computing, 2020.

RETSLAFF, F. A. S.; FIGUEIREDO FILHO, A.;, DIAS, A. N.; BERNETT, L. G.; FIGURA, M. A. . Curvas de sítio e relações hipsométricas para Eucalyptus grandis na região dos Campos Gerais, Paraná. *CERNE*, Lavras , v.21, n.2, p.219-225, 2015.

SANTOS, A.; SILVA, S.; LEITE, H.; CRUZ, J. Influência da variabilidade edafoclimática no crescimento de clones de eucalipto no Nordeste baiano. *Pesquisa Florestal Brasileira*, v.37, n.91, p.259-268, 2017.

SCOLFORO, J. R. S. *Biometria florestal: Modelos de crescimento e produção florestal*. Lavras: Editora UFLA/FAEPE, 2006. 393p.

SILVA, É. M. da; TADEU, M. H.; SILVA, V. P. da; PIO, R.; FERNANDES, T. J.; MUNIZ, J. A. Description of blackberry fruit growth by nonlinear regression models. *Revista Brasileira de Fruticultura*, v.42, n.2, e-177, 2020.

SHAPIRO, S. S.; WILK, M. B. An analysis of variance test for normality. *Biometrika*, London, v.52, n.3/4, p.591-611, 1965.

VENDRUSCOLO, D. G. S.; DRESCHER, R.; SOUZA, H. S.; MOURA, J. P. V. M.; MAMORÉ, F. M. D.; SIQUEIRA, T. A. da S. Estimativa da altura de eucalipto por meio de regressão não linear e redes neurais artificiais. *Revista Brasileira de Biometria*, São Paulo, v.33, n.4, p.556-569, 2015.

Received on 31.08.2020.

Approved after revised on 28.01.2021.