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ARTICLE

Study of the growth of *Amaranthus* weeds using non-linear models

Edilene Cristina Pedroso Azarias*[1], Edilson Marcelino Silva [2], Joel Augusto Muniz [3] and Tales Jesus Fernandes [3]

[1]Postgraduate program in statistics and agricultural experimentation, Department of Statistics, Federal University of Lavras, Lavras - MG, Brazil
[2]Mathematics department, Federal Rural University of Rio de Janeiro, Seropédica - RJ, Brazil
[3] Department of Statistics, Federal University of Lavras, Lavras - MG, Brazil
*Corresponding author. Email: edilene.cris100@hotmail.com

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Abstract
There are several known weed species, especially the genus *Amaranthus*. They have rapid spread and growth, compete for water, light, and nutrients, and harm crops. Studying the growth of these plants allows to analyze their interference and contributes to the development of management techniques. The Logistic and Gompertz non-linear models were fit using the R software to the growth data of five *Amaranthus* weed species, with evaluations at 20, 30, 40, 50, 60, 70, 80, and 90 days after sowing, considering the autoregressive error structure (AR1) and heteroscedasticity of variances. The critical points of the fitted curves were analyzed and the best model for data description was evaluated. Models were evaluated by R^2, AIC, DPR, and Bates and Watts curvatures. For the root dry mass, the Gompertz model was the best for *A. deflexus*, *A. hybridus*, *A. retroflexus*, and *A. spinosus*, and the Logistic model for *A. viridis*. *A. deflexus* and *A. hybridus* presented the lowest and the highest maximum accumulations of total dry mass, respectively. *A. viridis* was the most precocious, in relation to root dry mass, *A. spinosus* was the latest.

Key words: Dry mass; Growth curve; Non-linear regression.

1. Introduction

The concept of weeds emerged with the beginning of agriculture, as from that moment, human beings began to select plants, determining their purposes, so those that grew among crops and were not of interest should be controlled so as not to harm others (Carvalho, 2013). There are several definitions for these types of plants, and the most adopted by the authors is related to their occurrence in sites where they interfere with the productivity of economic crops (Shaw, 1982; Vasconcelos *et al.* 2012). In this sense, according to Carvalho (2013), there is a relationship between some plants and humans, and this is what defines whether a given plant will be considered a weed or not.

The various interferences caused by weeds include an increase in production costs, as perhaps the producer will have to promote some management technique to combat them, reduction in the value of the place where they are, are hosts of pests, promote quality losses in production, and the development of other plants where they coexist can also be affected, as they need the same resources to develop, so perhaps there is competition, which can lead to reductions in agricultural production (Carvalho, 2013).

Weeds are present in several places around the world, including those of the genus *Amaranthus*, also known as caruru. According to Carvalho *et al.* (2006), in Brazil, *Amaranthus deflexus* (low amaranth), *Amaranthus hybridus* (smooth pigweed), *Amaranthus lividus* (livid pigweed), *Amaranthus retroflexus* (redroot pigweed), *Amaranthus spinosus* (spiny pigweed) and *Amaranthus viridis* (slender amaranth) are the most prominent.

Caruru belongs to the family Amaranthaceae, it is an annual, herbaceous plant, considered invasive and grows quickly, has high seed production, some species produce from 100,000 to 150,000 seeds and can remain for 5 to 10 years in the area (Carvalho *et al.*, 2006; Horak & Loughin, 2000). Some species of this genus are hosts of pests, and have an allelopathic effect, releasing chemical substances into the environment that can benefit or harm others, in addition to showing resistance to herbicides (Souza *et al.*, 2006).

Studies on these types of plants are very important since according to Carvalho *et al.* (2008), they allow to evaluate their behavior in the environment and their competitive ability, and can collaborate with the management system and the determination of an adequate period for herbicide applications (Sellers *et al.*, 2003). Plant growth is usually described by non-linear models, as it occurs in phases and has a sigmoidal pattern, at the beginning it is slow, but later the development of the root system and the emergence of leaves begins, in this phase, there is rapid growth and after some time it reaches the senescence phase where there is a decrease in leaves and light energy absorption resulting in a lower accumulation of dry matter (Peixoto & Peixoto, 2009; Jane *et al.*, 2019).

Several studies used non-linear models to describe growth such as the height of *Eucalyptus* hybrid clones (Frühauf *et al.*, 2022), height of bean plants (Frühauf *et al.*, 2021), wood volume of *Eucalyptus urophylla* x *Eucalyptus grandis* (Silva *et al.*, 2021b), in sugarcane varieties (Jane *et al.*, 2020) and peppers of the Doce cultivar (Jane *et al.*, 2019). Curves of the models used to describe growth, in general, have a sigmoidal shape, and their parameters have a practical interpretation. The most used models according to Fernandes *et al.* (2015) are Logistic, Gompertz, von Bertalanffy, Brody and Richards.

When studying these models, some points on the growth curve have to be analyzed such as maximum acceleration (MAP), maximum deceleration (MDP), asymptotic deceleration (ADP), and inflection (IP), as they can also help professionals in the area about the management of these weeds and can be determined by studying higher order derivatives. These critical points were studied in the growth of lettuce (Carini *et al.*, 2020), pout pepper (Diel *et al.*, 2020), coconut fruit (Silva *et al.*, 2021a), and Campolina horses (Teixeira *et al.*, 2021).

Non-linear regression analysis was also adopted by some authors to check the interference of weeds in some crops, such as in studies on the interference of slender amaranth weeds with sugar beet growth and production characteristics (Marcolini *et al.*, 2010), in the verification of the leaf nutrient content in weeds and coffee plants grown in competition (Fialho *et al.*, 2012), in the susceptibility of five *Amaranthus* weed species to post-emergence herbicides (Carvalho *et al.*, 2006) and to verify the resistance of *A. retroflexus* to acetolactate synthase-inhibiting herbicides in cotton (Francischini *et al.* 2014).

In this context, this study aimed to analyze the growth of *Amaranthus* weeds, in non-competitive conditions, based on the accumulation of total dry mass, root dry mass, and reproductive structures dry mass, using non-linear models Logistic and Gompertz and explore the derivatives from first to fourth order, and analyzing the critical points of the growth curves. In addition, also determined the most appropriate model to describe the growth of the species, providing practical and relevant information for researchers in the area.

2. Material and Methods

The data analyzed here were extracted from Carvalho *et al.* (2008), the experiment was carried out in a greenhouse at the Department of Plant Production of the Agriculture School "Luiz de Queiroz" – ESALQ/USP, between September and December 2005. Seeds of *A. hybridus* (smooth pigweed), *A. retroflexus* (redroot pigweed), and *A. viridis* (slender amaranth); fruit of *A. deflexus* (low amaranth) and *A. spinosus* (spiny pigweed) were used, according to the form of dispersal of each.

The seeds or fruits were allowed to germinate in 2 L plastic boxes, with commercial substrate (Pine bark + peat + vermiculite). After, they were transplanted into pots where they remained until the end of the experiment. The experimental plots were plastic pots with a capacity of 2.8 L, with a mixture of commercial substrate and vermiculite, in the proportion of 2:1, respectively, and were irrigated when necessary.

This was a randomized block experimental design with three replications, evaluating the five weed species. In each species, eight evaluations were carried out for growth, at the ages of 20, 30, 40, 50, 60, 70, 80, and 90 days after sowing (DAS). For each evaluation, three plants were randomly sampled and taken to the laboratory for analysis by destructive process, which characterizes the cross-sectional data analysis method. Plants were washed in running water and had their variables analyzed. The sampled material was oven-dried at 70ºC for 72 hours, and the dry mass (g $plant^{-1}$) of roots, branches, leaves, inflorescences (flowers + fruits) and the total dry mass was measured.

The Logistic and Gompertz non-linear models were fit, considering x_i the ages in days after sowing (DAS) of the plants, Y_i the accumulation of total dry mass, in the roots and reproductive structures for the average of the three plants. These models were chosen because they are the most used in the literature to describe the growth of living beings, mainly in agriculture (Jane *et al.*, 2020). The parameterization considered was based on Fernandes *et al.* (2015), the non-linear models used were the Logistic $Y_i = \frac{\alpha}{1+e^{k(\beta-x_i)}} + \varepsilon_i$ and Gompertz $Y_i = \alpha e^{-e^{k(\beta-x_i)}} + \varepsilon_i$, where: i = 1,2, ..., n. Where n is the number of times the evaluations were performed, Y_i is the i-th observation of the dependent variable; x_i is the i-th observation of the independent variable; α represents the horizontal asymptote, that is, it is the expected value for the maximum growth of the object under study, when $x_i \to \infty$; β is the abscissa of the inflection point, from this point the growth is decelerated; k is an index associated with growth or maturity, the higher its value, the less time it will take for the object of study to reach the inflection point; ε_i is the random errors attributed to the model, independent and identically distributed with normal distribution of zero mean and constant variance (σ^2), that is, $\varepsilon_i \sim N(0;\sigma^2)$.

The estimation of the model parameters was based on the method of least squares, using the Gauss-Newton algorithm. For analysis of the residuals, the Shapiro-Wilk, Breusch-Pagan, and Durbin-Watson tests were applied to check assumptions of normality, homoscedasticity, and independence of the residuals, respectively.

At first, the method of ordinary least squares was adopted to estimate the parameters, taking into account the assumptions about the residuals such as independence, normality, and homoscedasticity as met. With the adjustments made, residuals were analyzed and when the assumption of normality was met, the confidence intervals were constructed for the parameters α, β, and k. Regarding dependence of residuals, if it occurred, the models were fit again with the inclusion of the first-order autoregressive term (AR1). In case of heteroscedasticity, the weighting factor was estimated, incorporating the existing uncertainty about each factor, using the "weights" argument, and the following functions "varIdent()", "varExp()", "varPower() ", "varConstPower()", "varConstProp()", from the "gnls" function using the R software (R Core Team, 2023). To compare to the other fit models, the weighted model with the lowest value of the Akaike Information Criterion (AIC) was selected.

Student's t-test was applied to check the significance of parameters α, β, k. In cases where there was residual autocorrelation, the $\emptyset$ parameter was included. The null hypothesis (H_o) is that the parameter is statistically equal to zero and the alternative hypothesis (H_a) that the parameter is statistically different from zero. The 95% confidence intervals were also obtained.

After fitting the models, the comparison and selection of the model that best described the data were made based on the goodness of fit meansures: coefficient of determination (R^2), residual standard deviation (RSD), Akaike Information Criterion (AIC), and Bates and Watts curvatures, which measure the non-linearity of the model, with the model that presented the highest R^2 value and lowest values for the DPR, AIC and parametric (C^{θ}) and intrinsic (C^{ι}) non-linearity being considered the most appropriate.

First to fourth-order derivatives in relation to time, from the Logistic and Gompertz models, were used to find the critical points: point of maximum acceleration (MAP), point of inflection (IP), point of maximum deceleration (PMD) and point of asymptotic deceleration (PAD), and the curve of the first derivative was plotted. The coordinates of the points referring to the models that best fit the data were calculated according to Silva *et al.* (2021a).

The estimation of model parameters, statistical tests, graphs, residual analysis, confidence intervals, and verification of the goodness of fit for the selection of models in this study were performed using free R software (R Core Team, 2023). The packages used were "*nlme*" (Pinheiro *et al.*, 2023), "*car*" (John & Sanford, 2023), "lmtest" (Achim & Hothorn, 2022). The significance level adopted for the tests was 5%.

3. Results and Discussion

Initially, the ordinary least squares method was used for fitting the two models to the data, subsequently, residual analysis was carried out to check the assumptions about the error vector, that is, whether they are independent, identically distributed with normal distribution with mean zero and constant variance, using the Shapiro-Wilk, Durbin-Watson and Breusch-Pagan tests.

In Table 1, the assumption of normality by the Shapiro-Wilk test was met in all models for all species, that is, p-value > 0.05. The assumption of homogeneity of the variances of the residuals, using the Breusch-Pagan test, in the Gompertz model for *A. retroflexus*, was not met (p-value < 0.05), indicating the presence of heteroscedasticity of residual variances.

Table 1. P-values of the Breusch-Pagan, Shapiro-Wilk, and Durbin-Watson tests applied to residuals of the Logistic and Gompertz models fit for root dry mass in weeds *A. deflexus*, *A. hybridus*, *A. retroflexus*, *A. spinosus*, and *A. viridis*

Species	Model	Shapiro-Wilk	Breusch-Pagan	Durbin-Watson
A. deflexus	Logistic	0.2527	0.4040	0.9960
	Gompertz	0.1266	0.6194	0.5140
A. hybridus	Logistic	0.5033	0.5774	0.0000**
	Gompertz	0.9785	0.0840	0.2860
A. retroflexus	Logistic	0.3887	0.0608	0.1940
	Gompertz	0.7204	0.0286*	0.3640
A. spinosus	Logistic	0.3191	0.6909	0.0440*
	Gompertz	0.4575	0.2415	0.9220
A. viridis	Logistic	0.8742	0.3378	0.9480
	Gompertz	0.8244	0.4048	0.7720

* significant at the 5% probability level

Given the heteroscedasticity of variances, the parameters need to be estimated by weighting. The "weights" argument of the gnls function of the nlme package (Pinheiro *et al.*, 2023) in the R software, which uses this approach, and the class that best fit to the root dry mass data in the roots was "VarExp ()", "Exponential variance function", where the weights are calculated based on an exponential function. The assumption of homogeneity of variances was also violated in the study by Silva (2021), on carbon dynamics in soil treated with tannery sludge. According to Fernandes *et al.* (2014), who compared the fit of the Logistic and Gompertz models in the description of the growth curves of the coffee bean, when the analysis involves growth, it is common to observe heterogeneous variance, because, with the passage of time and the development of plants, such as the example of this study, there is a greater variation in size.

With the Durbin-Watson test, in the fit of the Logistic model for the species *A. hybridus* and *A. spinosus*, there was autocorrelation in the residuals (p-value < 0.05). Thus, the fit was performed by the generalized least squares method, with the inclusion of the first-order autoregressive term. Autocorrelated errors were also verified by Frühauf *et al.* (2022), Silva *et al.* (2021a), and Frühauf *et al.* (2020), when fitting non-linear models to height data of hybrid clones of *Eucalyptus*, green dwarf coconut, and diametric growth of cedar, respectively.

Most of the fits did not present a correlation of the residuals, this may be because the data used here were cross-sectional, that is, the determinations made at each age in the laboratory were carried out in a destructive way, so the data came from different plants. This evaluation method was also used by Patrianova *et al.* (2010), Ribeiro *et al.* (2018), Souza *et al.* (2019), and Teixeira *et al.* (2021).

Table 2 lists the results obtained for the application of the Shapiro-Wilk, Breusch-Pagan, and

Durbin-Watson tests for the dry mass of reproductive structures of the studied weed species.

Table 2. P-values of the Breusch-Pagan, Shapiro-Wilk, and Durbin-Watson tests applied to residuals of the Logistic and Gompertz models fit for the dry mass of reproductive structures of *A. deflexus*, *A. hybridus*, *A. retroflexus*, *A. spinosus*, and *A. viridis*

Species	Model	Shapiro-Wilk	Breusch-Pagan	Durbin-Watson
A. deflexus	Logistic	0.2985	0.4940	0.9761
	Gompertz	0.5055	0.4226	0.0480*
A. hybridus	Logistic	0.8874	0.3100	0.4420
	Gompertz	0.1965	0.7089	0.7260
A. retroflexus	Logistic	0.9672	0.1711	0.9720
	Gompertz	0.6144	0.2726	0.8600
A. spinosus	Logistic	0.6100	0.7435	0.9880
	Gompertz	0.9716	0.2620	0.3140
A. viridis	Logistic	0.3944	0.7467	0.2540
	Gompertz	0.1929	0.7940	0.6780

* significant at the 5% probability level

Based on the Shapiro-Wilk and Breusch-Pagan tests for normality and homogeneity of variances of the residuals, respectively, the assumptions were met, that is, p-value > 0.05. By the Durbin-Watson test, for the Gompertz model in *A. deflexus*, the residuals showed autocorrelation (p-value < 0.05). Thus, the fit was made again incorporating the first-order autoregressive parameter.

Table 3 lists the results obtained from the application of the Shapiro-Wilk, Breusch-Pagan, and Durbin-Watson tests on the total dry mass of the studied weeds.

Table 3. P-values of the Breusch-Pagan, Shapiro-Wilk, and Durbin-Watson tests applied to residuals of the Logistic and Gompertz models fit for the total dry mass of weed *A. deflexus*, *A. hybridus*, *A. retroflexus*, *A. spinosus*, and *A. viridis*

Species	Model	Shapiro-Wilk	Breusch-Pagan	Durbin-Watson
A. deflexus	Logistic	0.1244	0.2547	0.8240
	Gompertz	0.7126	0.5031	0.7360
A. hybridus	Logistic	0.7405	0.6326	0.2400
	Gompertz	0.1940	0.2554	0.6860
A. retroflexus	Logistic	0.4251	0.6393	0.3080
	Gompertz	0.7907	0.0449*	0.0560
A. spinosus	Logistic	0.1298	0.3633	0.3100
	Gompertz	0.5894	0.1276	0.8740
A. viridis	Logistic	0.2362	0.3101	0.1120
	Gompertz	0.9033	0.0426*	0.3260

* significant at the 5% probability level

According to the results presented, assumptions of normality and independence of residuals based on the Shapiro-Wilk and Durbin-Watson tests, respectively, were met, that is, p-value > 0.05.

The Breusch-Pagan test showed that the assumption of homogeneity of variances of the residuals for the Gompertz model in *A. viridis* and *A. retroflexus* was not met (p-value < 0.05), indicating that there was heteroscedasticity of variances of residuals. Regarding the tested classes, "varPower ()", and "Power variance function", which presents a power variance function structure, was the best-fit variable for both species.

Table 4 shows the goodness-of-fit evaluators of the two models for root dry mass, both fitted well with the data, but Gompertz presented the lowest values of AIC, RSD, and measures of parametric (C^{θ}) an intrinsic (C^{ι}), non-linearity, in addition to a higher R^2 value in *A. deflexus*, *A. hybridus*, *A. retroflexus,* and *A. spinosus*. In the roots, the accumulation of dry mass at the inflection point was lower in the studied species than in reproductive structures and total dry mass, since the Gompertz model presented a lower inflection point ordinate than the Logistic model. This can be seen in Figure 2. While the Logistic model exhibited a better fit in *A. viridis*, considering the AIC, RSD, and R^2 values, since there was no convergence for the non-linearity measures, for the Gompertz model. Chiapinotto *et al.* (2017) studied weed development at 7, 14, 21, and 28 days after herbicide application. The Logistic model was fit to the data and there was evidence of different and high levels of plant resistance.

Table 4. Evaluators of the goodness-of-fit: Akaike information criterion (AIC), residual standard deviation (RSD), coefficient of determination (R²), parametric non-linearity (c^{θ}), and intrinsic non-linearity (c^{ι}) for comparison of the Logistic and Gompertz models to root dry mass data

Species	Model	AIC	RSD	R²	C^{θ}	C^{ι}
A. deflexus	Logistic	-27.5955	0.0309	0.9986	0.3151	0.1354
	Gompertz	-53.1240	0.0067	0.9998	0.0861	0.0395
A. hybridus	Logistic	3.4292	1.2727	0.9223	0.4165	0.1306
	Gompertz	-29.6323	0.0291	0.9999	0.0704	0.0177
A. retroflexus	Logistic	0.0175	0.1858	0.9983	0.2682	0.1335
	Gompertz	−25.8021	0.2410	0.9995	0.1383	0.0518
A. spinosus	Logistic	-0.5485	0.2949	0.9929	0.3081	0.1317
	Gompertz	-17.2186	0.0633	0.9998	0.1649	0.0584
A. viridis	Logistic	-43.2599	0.0172	0.9998	0.0916	0.0003
	Gompertz	-41.7398	0.0187	0.9996		

Table 5 lists the estimates of parameters of the Logistic and Gompertz models, which offered better fits according to each species and goodness-of-fit evaluators, and their respective 95% confidence intervals for root dry mass.

Table 5. Estimates for parameters of the Logistic (*A. viridis*) and Gompertz (*A. deflexus, A. hybridus, A. retroflexus,* and *A. spinosus*) models and their respective confidence intervals, lower limit (LL) and upper limit (UL), fit for root dry mass (g $plant^{-1}$)

Model	Species	Parameter	LL	Estimate	UL
Gompertz	*A. deflexus*	α	1.6798	1.6872	1.6946
		β	40.0562	40.1730	40.2901
		K	0.1748	0.1811	0.1874
	A. hybridus	α	11.7449	11.8200	11.8859
		β	40.7553	40.9300	41.1075
		K	0.0611	0.0622	0.0632
	A. retroflexus	α	9.3438	9.3595	9.3761
		β	38.1852	38.4630	38.7413
		K	0.1171	0.1211	0.1251
	A. spinosus	α	8.5701	8.6880	8.8059
		β	43.5142	43.8951	44.2761
		K	0.0804	0.0846	0.0888
Logistic	*A. viridis*	α	6.7313	6.7451	6.7588
		β	32.3399	32.3839	32.4278
		K	1.2410	1.2723	1.3037

Considering the Student's t-test, all parameters were significant, in addition, the confidence intervals did not contain zero, which indicates the adequacy of the models when describing the data of root dry mass accumulation in relation to time.

The graph of the confidence intervals for the α parameter, obtained by the R software (Figure 1), indicates no overlapping for the dry matter accumulation of the four species. *A. hybridus* had the highest accumulation, with 11.8200 $g\ plant^{-1}$, followed by *A. retroflexus* with 9.3595 $g\ plant^{-1}$, A. spinosus with 8.6880 $g\ plant^{-1}$, *A. viridis* with 6.7451 $g\ plant^{-1}$, and *A. deflexus* with 1.6872 $g\ plant^{-1}$.

A. retroflexus had a lower inflection point (β), indicating that its roots grew earlier than the others, and a higher value of k, that is, faster growth, which may represent an advantage in the use of resources from the environment (Carvalho *et al.* 2008), and as it is a weed, this deserves attention. *A. spinosus*, on the other hand, presented the highest value for the β parameter, indicating that it is later than the other species. *A. hybridus* exhibited a lower growth rate (k) and, as previously observed, showed a greater accumulation of root dry mass, but at a slower pace.

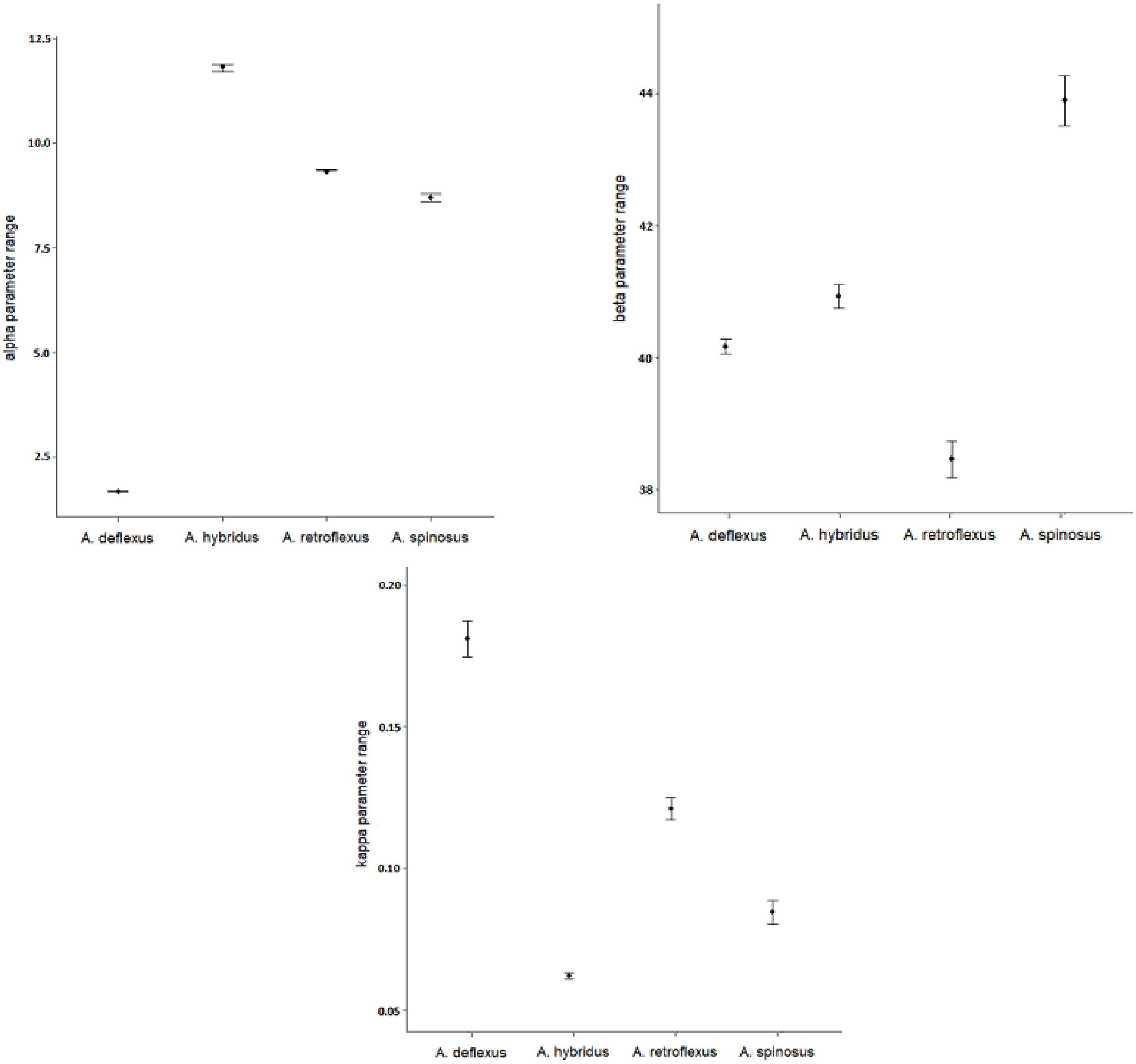

Figure 1. Confidence intervals for parameters α, β, and k, fit for the Gompertz model (*A. deflexus, A. hybridus, A. retroflexus,* and *A. spinosus*) for root dry mass.

Table 6 lists the critical points of the curves using the parameter estimates based on Silva *et al.* (2021a): maximum acceleration point (MAP), inflection point (IP), maximum deceleration point (MDP), and asymptotic deceleration point (ADP) for root dry mass. These critical points were also analyzed in the growth of eggplant (Sari *et al.*, 2018), lettuce (Carini *et al.*, 2020), pout pepper (Diel *et al.*, 2020), coconut fruit (Silva *et al.*, 2021a), and Campolina horses (Teixeira *et al.*, 2021).

Table 6. Critical points: maximum acceleration point (MAP), inflection point (IP), maximum deceleration point (MDP), and asymptotic deceleration point (ADP), for root dry mass for the Gompertz model (*A. deflexus*, *A. hybridus*, *A. retroflexus*, and *A. spinosus*), and for the Logistic model (*A. viridis*)

Model	Species	Point	MAP	IP	MDP	ADP
Gompertz	*A. deflexus*	Abscissa	34.8618	40.1732	45.4845	50.0975
		Ordinate	0.1231	0.6207	1.1526	1.4295
	A. hybridus	Abscissa	25.4138	40.9300	56.4461	69.9219
		Ordinate	0.8622	4.3483	8.0752	10.0151
	A. retroflexus	Abscissa	30.5218	38.4630	46.4043	53.3012
		Ordinate	0.6828	3.4432	6.3942	7.9303
	A. spinosus	Abscissa	32.5260	43.8951	55.2643	65.1384
		Ordinate	0.6338	3.1961	5.9355	7.3614
Logistic	*A. viridis*	Abscissa	31.3472	32.3839	33.4205	34.1856
		Ordinate	1.4254	3.3725	5.3197	6.1263

Sari *et al.* (2018) proved the importance of these points, according to the authors, they are important to determine the productive performance of crops. According to Mischan and Pinho (2014), for the Logistic model, the MAP, IP, MDP, and ADP occur at approximately 21.1; 50.0; 78.8, and 90.8% of the upper asymptote, respectively. In the Gompertz model, however, they occur at approximately 7.29; 36.7; 68.25 and 84.73%, from the upper asymptote, respectively. The most accelerated and decrease growth phase is between the MAP and MDP, where the growth is most concentrated, in the Logistics model it occurs at 57.7%, in Gompertz at 60.96% of the growth (Sari *et al.*, 2018).

In *A. viridis*, the MAP and MDP abscissas were approximately 31 and 33 days, lower than the others, which is considered the ideal period to take some control measures. This species also presented an anterior MDP abscissa, which indicates a shorter life cycle with a senescence phase at approximately 34 DAS and a maximum dry mass accumulation of 6.1263 $g\ plant^{-1}$ in the roots, confirming the results presented in Table 5 where it showed accelerated growth.

A. hybridus exhibited MAP and MDP abscissas at approximately 25 and 56 days, respectively, superior to the others. As observed in Table 5, it also presented a longer cycle, approximately 70 days, and a maximum dry mass accumulation of 10.0151 $g\ plant^{-1}$, higher than that of Camalote grass (Carvalho *et al.*, 2005a) and white grass (Carvalho *et al.*, 2005b). In turn, *A. deflexus* presented lower MDP values, approximately at 50 DAS with a maximum dry mass accumulation of 1,4295 $g\ plant^{-1}$. MAP, IP, MDP, and ADP are represented in Figures 2 e 3, showing the growth rate curve. It should be noted that this rate is illustrative and only follows the x-axis.

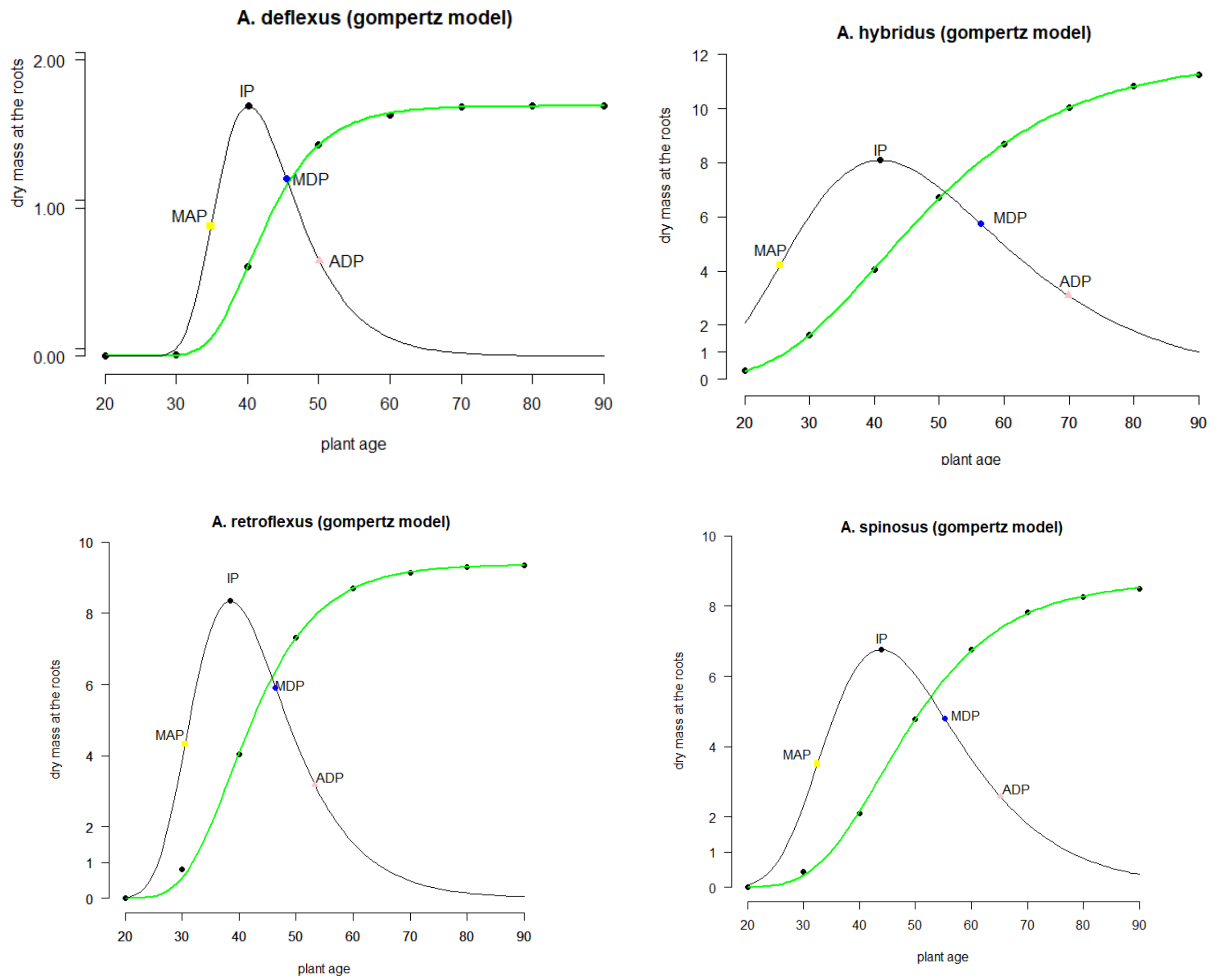

Figure 2. Growth rate with the inflection point (IP), maximum acceleration point (MAP), maximum deceleration point (MDP), asymptotic deceleration point (ADP), of the mean curves of the Gompertz model (*A. deflexus, A. hybridus, A. retroflexus,* and *A. spinosus*) fit to root dry mass data.

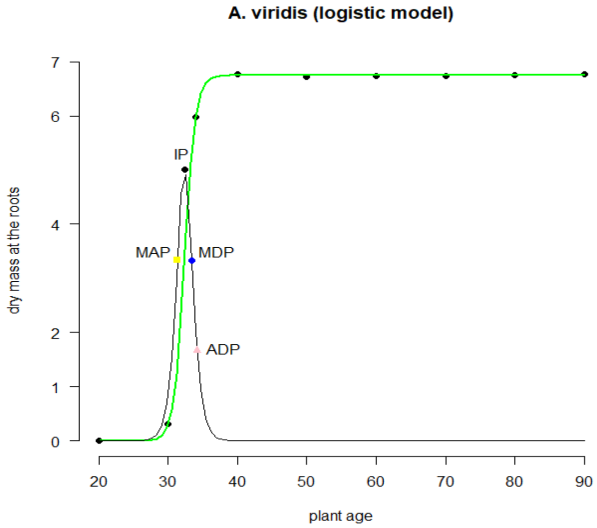

Figure 3. Growth rate with the inflection point (IP), maximum acceleration point (MAP), maximum deceleration point (MDP), asymptotic deceleration point (ADP), of the mean curves of the Logistic model (*A. viridis*), fit to root dry mass data.

According to the goodness-of-fit evaluators in Table 7, for *A. hybridus*, *A. retroflexus*, *A. spinosus*, and *A. viridis*, the Logistic model was the best fit. The Gompertz model fitted to data from *A. hybridus* and *A. retroflexus* showed a non-parametric curvature greater than 0.5, which is considered significant, indicating a deviation from linearity (Bates; Watts, 1980; Zeviani *et al.*, 2012; Fernandes *et al.*, 2015), with this model having lower AIC, RSD, measures of parametric (C^{θ}) and intrinsic (C^{ι}) non-linearity, and higher R^2, only for *A. deflexus*. Barroso *et al.* (2019) studied the growth and development of five weeds, in winter and summer, in relation to the number and dry mass of leaves, stems, roots, and total dry mass. They fitted the Logistic model to summer data and some winter variables. For the number of leaves and leaf dry matter, another non-linear model was fit, as it presents a different development pattern in summer.

Table 7. Goodness-of-fit evaluators: Akaike information criterion (AIC), residual standard deviation (RSD), coefficient of determination (R^2), parametric non-linearity (c^{θ}), and intrinsic non-linearity (c^{ι}) for comparison of the Logistic and Gompertz models to dry mass data in reproductive structures

Species	Model	AIC	RSD	R^2	C^{θ}	C^{ι}
A. deflexus	Logistic	1.6569	0.2035	0.9985	0.3088	0.1633
	Gompertz	-32.5357	0.0239	0.9998	0.0626	0.0295
A. hybridus	Logistic	3.7105	0.2356	0.9991	0.4258	0.1185
	Gompertz	-2.0783	0.1558	0.9996	0.5801	0.0902
A. retroflexus	Logistic	-1.3698	0.1639	0.9996	0.2185	0.0909
	Gompertz	3.4105	0.2306	0.9992	0.5674	0.2150
A. spinosus	Logistic	-9.2047	0.0937	0.9996	0.2621	0.0957
	Gompertz	-3.5450	0.1403	0.9992	0.4007	0.1084
A. viridis	Logistic	-1.0817	0.1673	0.9997	0.1422	0.0722
	Gompertz	7.3186	0.3049	0.9992	0.3675	0.1706

Table 8 presents the estimates of the model parameters and their 95% confidence intervals for the dry mass in reproductive structures of the studied weeds. Considering heterogeneous variance and adding the first-order autoregressive parameter AR (1) when necessary.

Table 8. Estimates for parameters of the Gompertz model (*A. deflexus*) and the Logistic model (*A. hybridus, A. retroflexus, A. spinosus,* and *A. viridis*) with first-order autoregressive error structure (AR1) and their respective confidence intervals, lower limit (LL) and upper limit (UL), fit for dry mass (g $plant^{-1}$) in reproductive structures

Model	Species	Parameters	LL	Estimate	UL
Gompertz	*A. deflexus*	α	9.5813	9.6021	9.6228
		β	51.1977	51.2457	51.2936
		K	0.1540	0.1563	0.1585
		$\emptyset$	-0.9631	-0.8027	-0.2220
Logistic	*A. hybridus*	α	15.6634	16.3067	16.9499
		β	66.4798	67.4856	68.4913
		K	0.1251	0.1397	0.1544
	A. retroflexus	α	14.3809	14.6772	14.9737
		β	63.6555	64.1695	64.6835
		K	0.1723	0.1867	0.2011
	A. spinosus	α	8.3281	8.4602	8.5922
		β	59.4011	59.7882	60.1753
		K	0.2124	0.2334	0.2543
	A. viridis	α	18.8693	19.1100	19.3506
		β	56.5041	56.8763	57.2484
		K	0.1726	0.1830	0.1934

Considering the Student's t-test, all parameters were significant and their confidence intervals did not contain zero, indicating that they were adequate to describe the accumulation of dry mass in reproductive structures in relation to time.

The ranges of the α and β parameter estimates did not overlap, as seen in Figure 4, for no species. *A. viridis* presented the highest α, that is, superior accumulation of dry mass in reproductive structures, while *A. spinosus* presented the lowest accumulation. The inflection point for *A. hybridus* occurred later than the others and in relation to the growth rate (k), this species exhibited a lower value, indicating that the onset of the reproduction cycle starts to slow down later, and more slowly compared to the other species.

The critical points for the dry mass in reproductive structures are listed in Table 9. *A. deflexus* presented lower values of MAP, IP, MDP, and ADP, the abscissa of MAP and MDP occurred at 45 and 57 days, respectively, indicating earlier cycles, and maximum dry mass accumulation of 8.1359 $g\ plant^{-1}$, which occurred approximately at 63 DAS, while for *A. viridis* at 69 DAS, it was 17.3569 $g\ plant^{-1}$. *A. spinosus* expressed the lowest maximum dry mass accumulation in reproductive structures, with ADP occurring at approximately 69 DAS with a maximum accumulation of 7.6841 $g\ plant^{-1}$. *A. deflexus* showed an inflection point earlier than the others, at approximately 51 DAS, with a dry mass accumulation that was also low, 3.5324 $g\ plant^{-1}$.

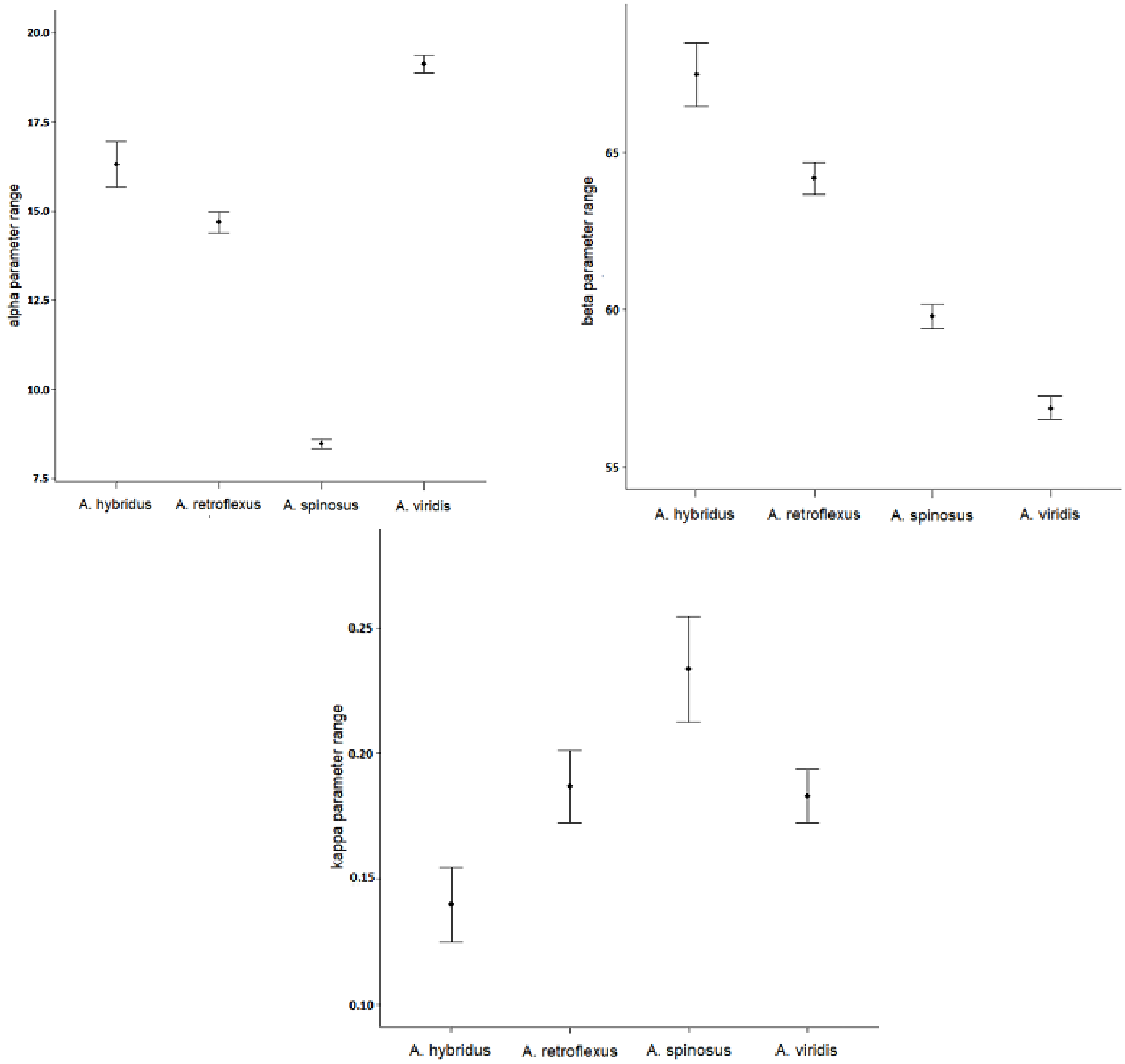

Figure 4. Confidence intervals for parameters α, β, and k, fit for the Logistic model (*A. hybridus, A. retroflexus, A. spinosus*, and *A. viridis*) for dry mass in reproductive structures.

Table 9. Critical points: maximum acceleration point (MAP), inflection point (IP), maximum deceleration point (MDP), and asymptotic deceleration point (ADP), for dry mass in reproductive structures for the Gompertz model (*A. deflexus*), and for the Logistic model (*A. retroflexus*, *A. spinosus*, *A. viridis*, and *A. hybridus*)

Model	Species	Point	MAP	IP	MDP	ADP
Gompertz	*A. deflexus*	Abscissa	45.0896	51.2457	57.4017	62.7482
		Ordinate	0.7004	3.5324	6.5599	8.1359
Logístico	*A. hybridus*	Abscissa	58.0452	67.4856	76.9259	83.8927
		Ordinate	3.4460	8.1533	12.8607	14.8108
	A. retroflexus	Abscissa	57.1059	64.1695	71.2331	76.4459
		Ordinate	3.1017	7.3386	11.5756	13.3309
	A. spinosus	Abscissa	54.1367	59.7882	65.4397	69.6104
		Ordinate	1.7878	4.2301	6.6723	7.6841
	A. viridis	Abscissa	49.6700	56.8763	64.0826	69.4007
		Ordinate	4.0384	9.5550	15.0716	17.3569

Figures 5 e 6 illustrates the growth rate curves, MAP, IP, MDP, and ADP, for the dry mass of reproductive structures.

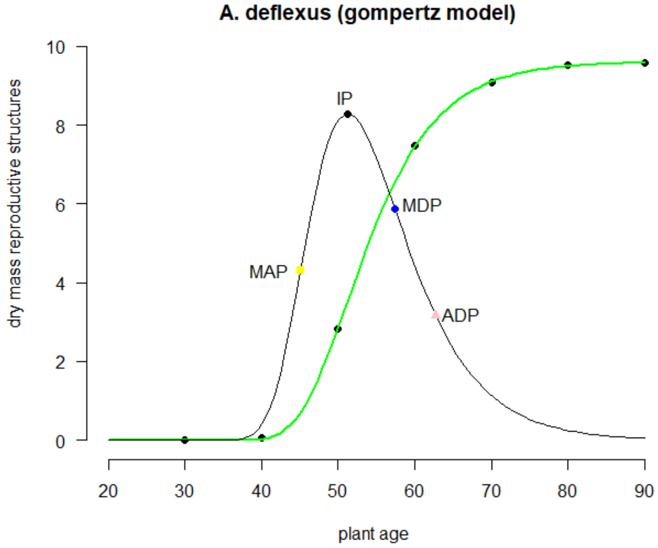

Figure 5. Growth rate with the inflection point (IP), maximum acceleration point (MAP), maximum deceleration point (MDP), and asymptotic deceleration point (ADP), of the mean curves of the Gompertz model (*A. deflexus*) fit to dry mass in reproductive structures.

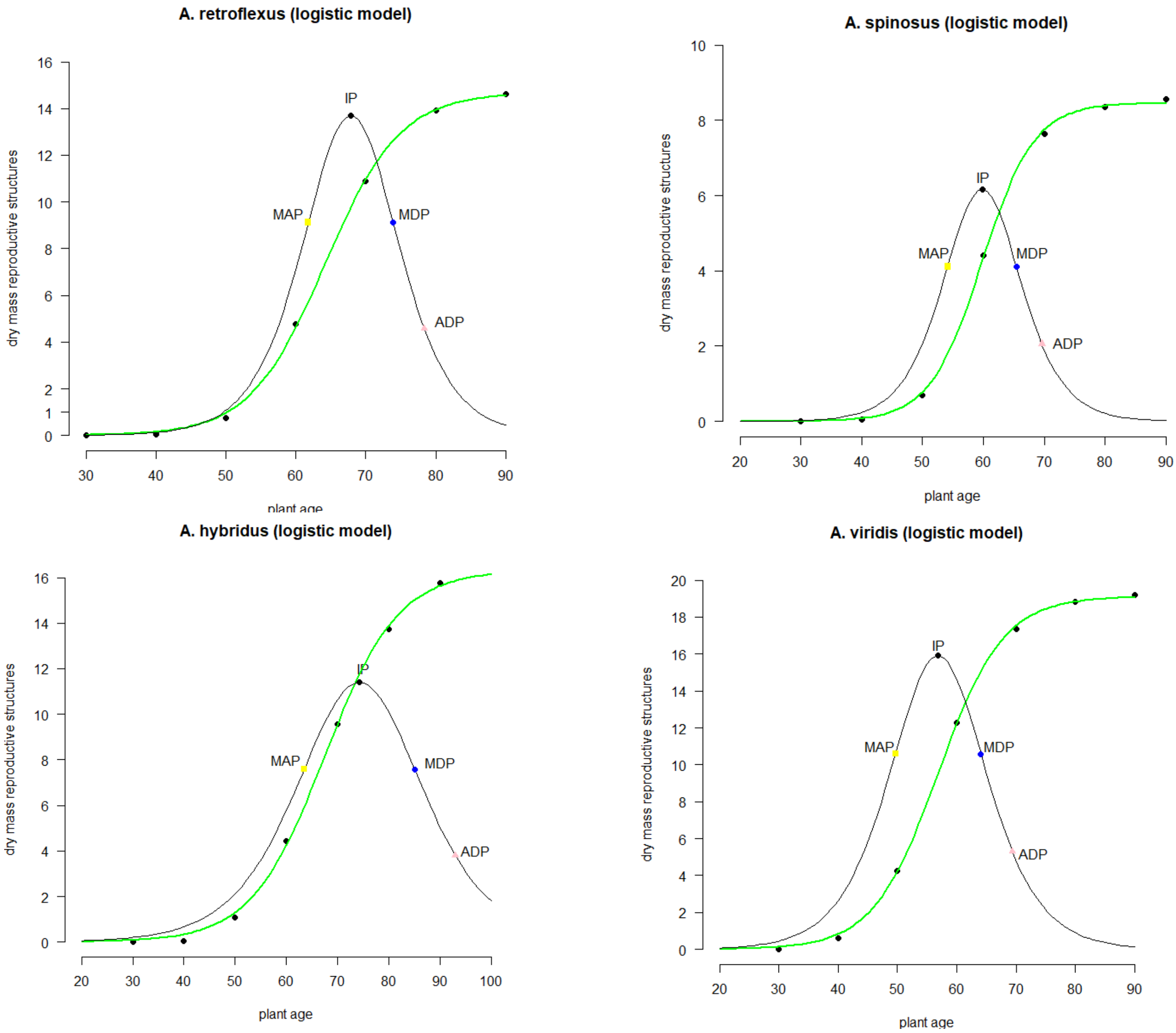

Figure 6. Growth rate with the inflection point (IP), maximum acceleration point (MAP), maximum deceleration point (MDP), and asymptotic deceleration point (ADP), of the mean curves of the Logistic model (*A. retroflexus*, *A. spinosus*, *A. viridis*, and *A. hybridus*) fit to dry mass in reproductive structures.

Table 10 presents the goodness-of-fit evaluators for total dry mass. The parametric and intrinsic non-linearity were not significant for all models in all species, and the Gompertz model showed the lowest AIC, RSD, lower measures of parametric (C^{θ}) and intrinsic (C^{ι}) non-linearity, and higher R^2 in *A. hybridus* and *A. retroflexus*, and the Logistic model for *A. deflexus* and *A. spinosus*. Considering the non-linearity measures for *A. viridis*, the Logistic model presented lower values in relation to the Gompertz model.

Table 10. Goodness-of-fit evaluators: Akaike information criterion (AIC), residual standard deviation (RSD), coefficient of determination (R^2), parametric non-linearity (c^{θ}), and intrinsic non-linearity (c^{ι}) for comparison of the Logistic and Gompertz models to total dry mass data

Species	Model	AIC	RSD	R^2	C^{θ}	C^{ι}
A. deflexus	Logistic	7.8088	0.3024	0.9996	0.1601	0.0735
	Gompertz	13.6757	0.4364	0.9992	0.2639	0.0981
A. hybridus	Logistic	28.0888	1.0742	0.9990	0.2466	0.1070
	Gompertz	19.6786	0.6351	0.9997	0.1978	0.0749
A. retroflexus	Logistic	33.5992	1.5159	0.9977	0.3455	0.1636
	Gompertz	4.5106	0.0782	0.9991	0.1211	0.0588
A. spinosus	Logistic	19.4175	0.6248	0.9994	0.1856	0.0830
	Gompertz	20.3833	0.6637	0.9994	0.2407	0.0836
A. viridis	Logistic	22.5218	0.7586	0.9989	0.1601	0.0735
	Gompertz	18.3387	0.1606	0.9989	0.2463	0.1017

Table 11 presents the estimates of the model parameters with 95% confidence intervals for the total dry mass of the studied weeds. All parameters were significant by Student's t-test, their confidence intervals did not contain zero, indicating that they were adequate to describe the accumulation of total dry mass in relation to time.

Table 11. Estimates for parameters of the Gompertz model (*A. hybridus* and *A. retroflexus*) and the Logistic model (*A. viridis, A. deflexus,* and *A. spinosus*) and their respective confidence intervals, lower limit (LL) and upper limit (UL), fit for total dry mass (g $plant^{-1}$)

Model	Species	Parameters	LL	Estimate	UL
Gompertz	*A. hybridus*	α	66.3307	67.4666	68.6025
		β	46.6517	47.1004	47.5490
		k	0.0904	0.0967	0.1030
	A. retroflexus	α	60.1638	62.7477	65.3316
		β	43.5801	44.3561	45.1321
		k	0.1042	0.1107	0.1171
Logistic	*A. viridis*	α	45.3274	46.1461	46.9649
		β	41.3463	41.9935	42.6406
		k	0.1663	0.1857	0.2052
	A. deflexus	α	27.7240	28.1063	28.4886
		β	49.4248	49.8797	50.3346
		k	0.1616	0.1736	0.1855
	A. spinosus	α	49.5420	50.3616	51.1812
		β	50.2027	50.7478	51.2929
		k	0.1537	0.1668	0.1798

The confidence intervals of the k parameter estimate, as observed in Figure 7, for the Logistic model intersected, which indicates no difference in the accumulation index for the total dry mass in these species.

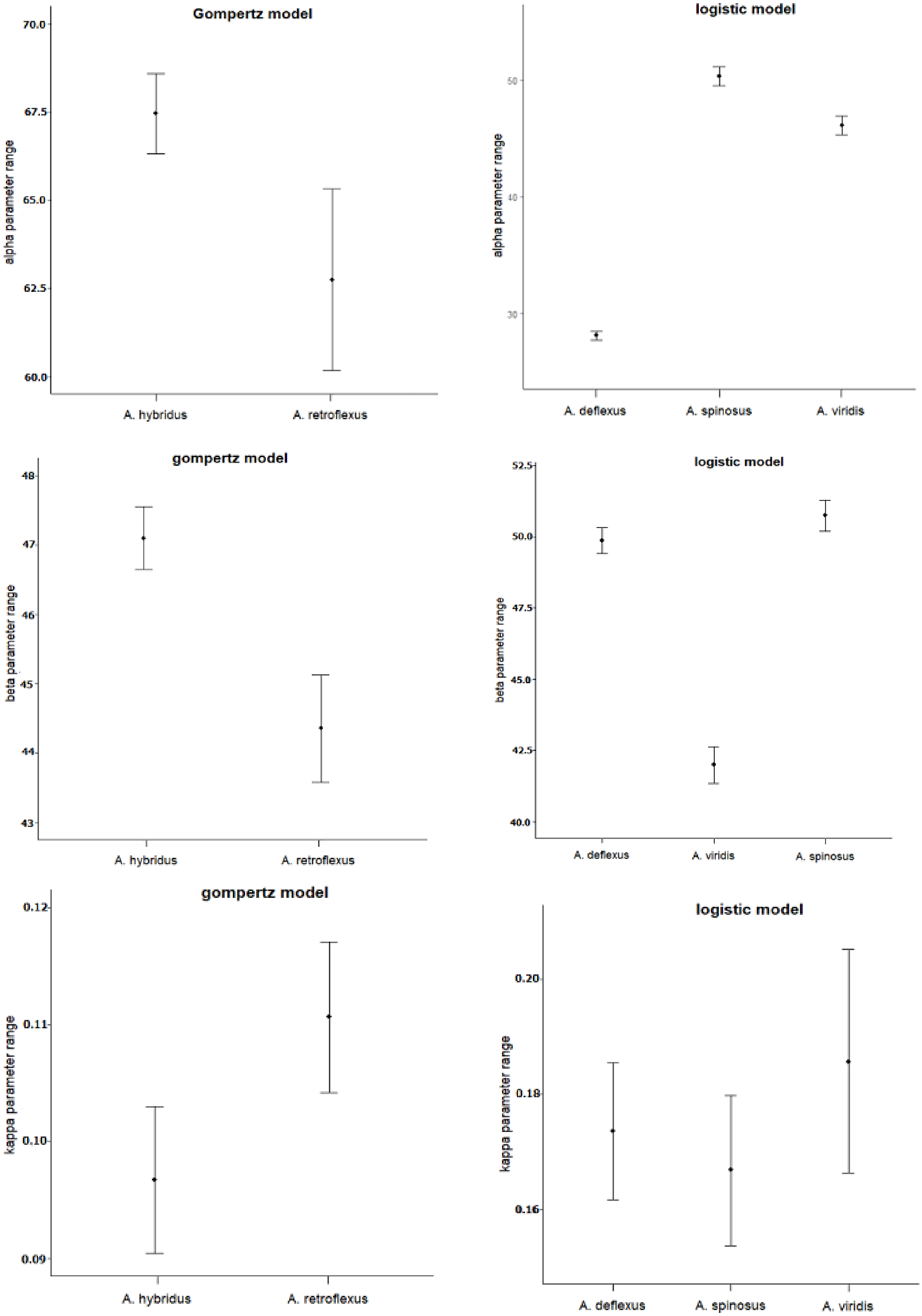

Figure 7. Confidence intervals for parameters α, β, and k, fit for the Gompertz model (*A. hybridus* and *A. retroflexus*) and for the Logistic model (*A. viridis, A. deflexus*, and *A. spinosus*) for total dry mass.

As seen in Table 11, *A. deflexus* and *A. hybridus* exhibited the lowest and highest accumulation of total dry mass, 28.1063 and 67.4666 g $plant^{-1}$, respectively. Regarding the inflection point (g $plant^{-1}$), the confidence intervals of *A. deflexus* and *A. spinosus* overlapped, indicating that they do not differ, and exhibited higher values for this parameter, that is, the growth of these species begins to slow down a little later than the others, approximately at 50 DAS. *A. viridis*, on the other hand, showed a lower value for the inflection point, which occurred approximately at 42 DAS, being this the most precocious species.

Considering the models that best described the data, the critical points of the curves were calculated using the values of the parameters found: maximum acceleration point (MAP), inflection point (IP), maximum deceleration point (MDP), and asymptotic deceleration point (ADP) for total dry mass, listed in Table 12.

Table 12. Critical points: maximum acceleration point (MAP), inflection point (IP), maximum deceleration point (MDP), and asymptotic deceleration point (ADP), for total dry mass for the Gompertz model (*A. hybridus* and *A. retroflexus*), and for the Logistic model (*A. deflexus*, A. *spinosus*, and *A. viridis*)

Model	Species	Point	MAP	IP	MDP	ADP
Gompertz	*A. hybridus*	Abscissa	37.1521	47.1004	57.0487	65.6888
		Ordinate	4.9216	24.8196	46.0917	57.1645
	A. retroflexus	Abscissa	35.6636	44.3561	53.0486	60.5981
		Ordinate	4.5773	23.0836	42.8678	53.1661
Logistic	*A. viridis*	Abscissa	34.8922	41.9935	49.0947	54.3353
		Ordinate	9.7518	23.0731	36.3946	41.9129
	A. deflexus	Abscissa	42.2811	49.8797	57.4783	63.0860
		Ordinate	5.9395	14.0531	22.1667	25.5280
	A. spinosus	Abscissa	42.8390	50.7478	58.6566	64.4932
		Ordinate	10.6427	25.1808	39.7190	45.7417

A. viridis presented MAP, IP, MDP, and ADP values earlier than the others, with a maximum dry mass accumulation of 41.9129 g $plant^{-1}$, approximately at 54 DAS, while at 63 DAS, *A. deflexus*, and at 66 DAS, *A. hybridus* exhibited maximum accumulation of 25.5280 and 57.1645 g $plant^{-1}$, respectively.

All weeds, except *A. deflexus*, reached values for total dry mass accumulation greater than *E. heterophylla*, *E. hyssopifolia*, and *E. hirta* studied by Ferreira *et al.* (2017), also to camalote grass (Carvalho *et al.*, 2005a), and *Digitaria insularis* (Machado *et al.*, 2006). The critical points for total dry mass in the studied weeds, according to the models that best described the data are represented in the growth rate curve, MAP, IP, MDP, and ADP (Figures 8 e 9).

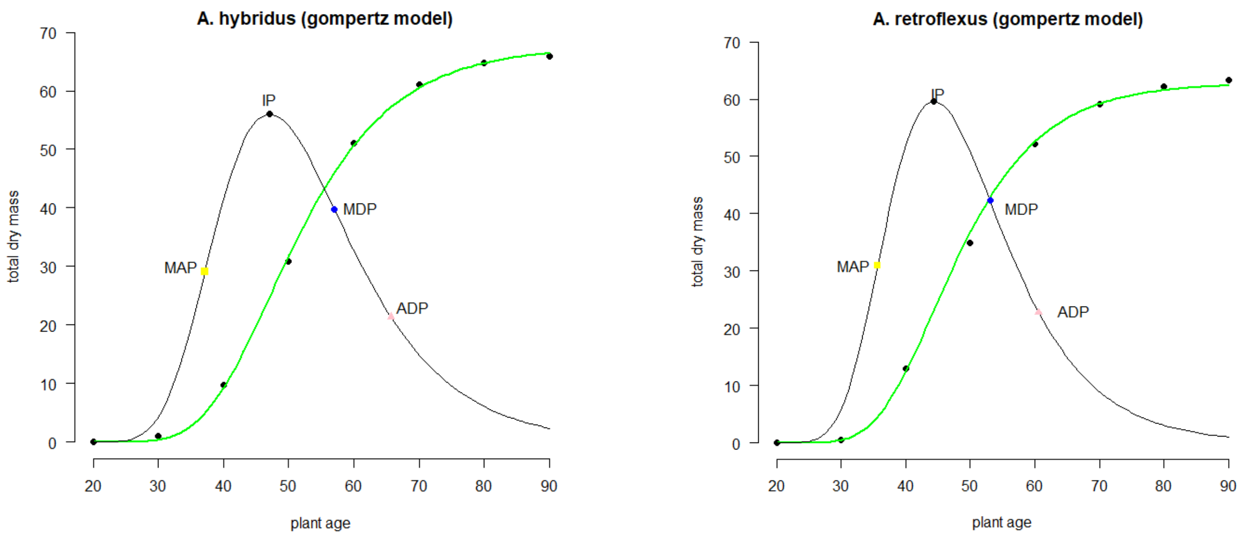

Figure 8. Growth rate with the inflection point (IP), maximum acceleration point (MAP), maximum deceleration point (MDP), asymptotic deceleration point (ADP), of the mean curves of the Gompertz model (*A. hybridus* and *A. retroflexus*).

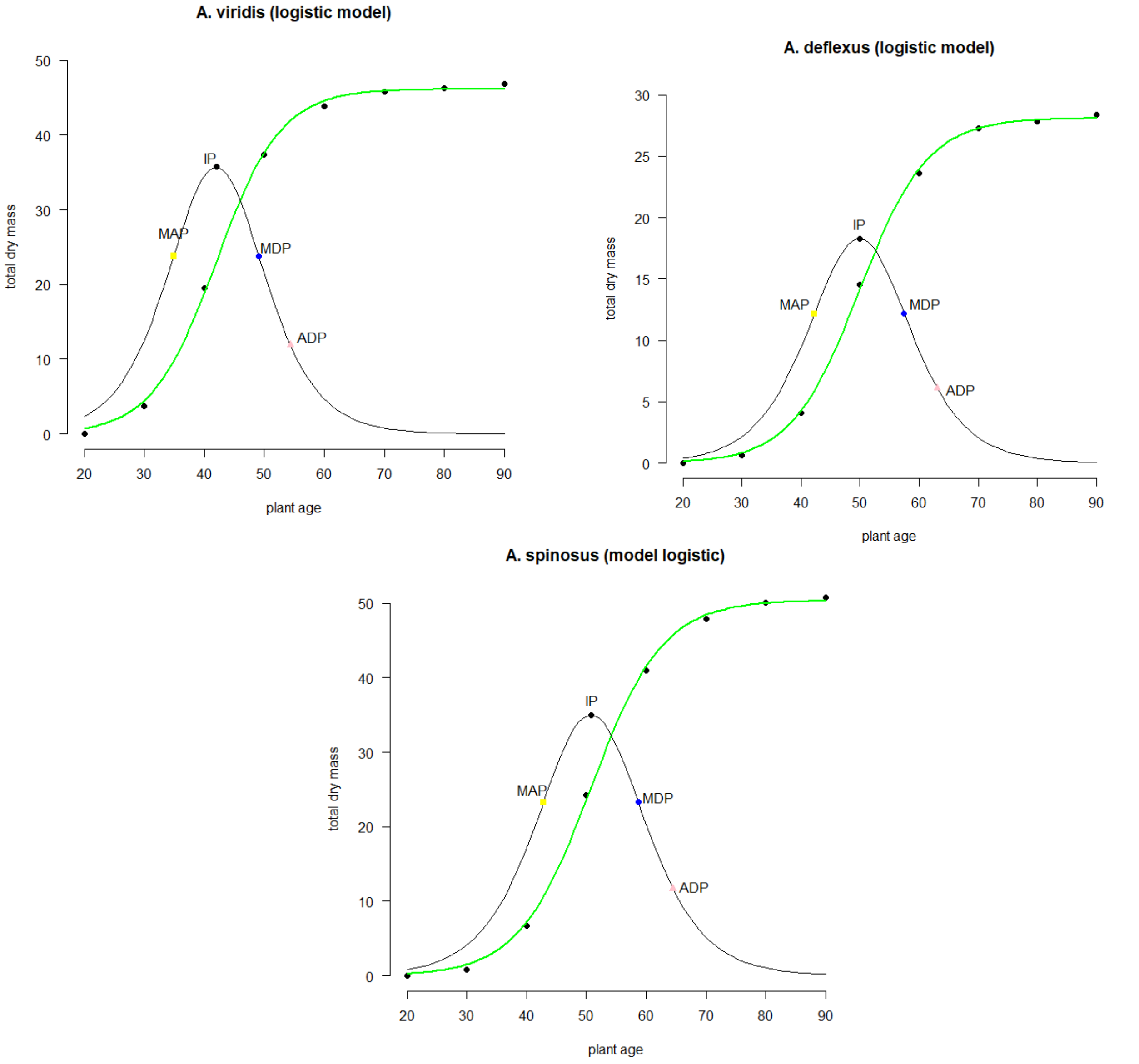

Figure 9. Growth rate with the inflection point (IP), maximum acceleration point (MAP), maximum deceleration point (MDP), asymptotic deceleration point (ADP), of the mean curves of the Logistic model (*A. deflexus*, A. *spinosus*, and *A. viridis*) fit to total dry mass data.

4. Conclusions

Regarding the root dry mass, the Gompertz model performed better in describing the dry mass accumulation in *A. deflexus*, *A. hybridus*, *A. retroflexus*, and *A. spinosus*, while the Logistic model showed a better fit in *A. viridis*.

As for reproductive structures, Gompertz was more appropriate for *A. deflexus*, and the Logistic model presented a better fit in *A. hybridus*, *A. retroflexus*, *A. spinosus*, and *A. viridis*.

For total dry mass, the Gompertz model presented a better fit in *A. hybridus* and *A. retroflexus*, and the Logistic model in *A. deflexus*, *A. spinosus*, and *A. viridis*.

The inflection points, for the root dry mass, indicated that *A. viridis* and *A. spinosus* were earlier and later, respectively. As for the dry mass in reproductive structures, *A. deflexus* was the most precocious, and *A. hybridus* was the latest weed. For the total dry mass, *A. viridis* obtained the lowest value for the inflection point, and *A. deflexus* and *A. spinosus* exhibited the lowest and highest total dry mass accumulation, respectively.

A. deflexus obtained the lowest maximum accumulation of total and root dry mass, of 1.4295 and 25.5280 g $plant^{-1}$, respectively. In the reproductive structures, *A. spinosus* showed the lowest accumulation, 7.6841 g $plant^{-1}$. On the other hand, *A. hybridus* exhibited the highest maximum accumulation of root dry mass, totaling 10.0151 and 57.1645 g $plant^{-1}$, respectively.

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Conflicts of Interest

The authors declare no conflict of interest.

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