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ARTICLE

Almost Unbiased Optimum Ratio-Type Estimator Population Mean in Stratified Sampling in Presence of Non-response

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Abstract
In this paper we have proposed classes of ratio-type estimators for finite population mean in presence of non-response in stratified sampling. The properties of the estimators have been discussed. We have also derived optimum choices of scalar constant to reduce the bias in the estimators which make suggested classes of estimators almost unbiased. The expressions of mean square error (MSE) have been derived up to the first order of approximation. We have compared our proposed classes of estimators with natural ratio estimators and with estimator under complete response. Results are also supported by empirical study.

Keywords: Mean square error; Ratio estimator; Study variable, Non-response, Stratified Sampling.

1. Introduction

Cochran (1977) introduced ratio estimator for estimating population mean when study variable and auxiliary variable are positively correlated. It is always been observed that the use of auxiliary information results in efficiency gain of the estimators over the estimators where no auxiliary information is used. Stratified sampling allows researchers in making the precision of the estimators better than the simple random sampling. So stratified sampling has often proved requisite in enhancing the precision of the estimators (Singh *et al.*, 2007).

Sometimes, in survey sampling, individuals who are selected for the survey are not able to take part in the survey which further leads to non-response problem. First time in literature non-response problem was examined by Hansen & Hurwitz (1946) in which they proposed a technique of taking a non-respondent's sub sample after first mail effort and that sub sample is enumerated by taking personal interview meeting. El-Badry (1956) further expanded Hansen-Hurwitz technique. Using the sub sampling procedure from non-respondents. Khare &

Srivastava (1993, 1997) proposed two phase sampling ratio and product estimator for the population mean and studied their properties. Kadilar (2003, 2005) suggested estimators in stratified sampling. Later Tabassum & Khan (2004) revisited the work of Khare & Srivastava (1993, 1997). Singh *et al*. (2008), Singh *et al*. (2009), Chaudhary *et al.* (2009), Chaudhary et al. (2011), Malik and Singh (2012), Sharma and Singh (2013), Malik and Singh (2013), Singh et al. (2014) and Sanaullah et al. (2015) have introduced some improved estimators in stratified random sampling. We have introduced almost unbiased optimum ratio type estimators (AUORE) in existence of non-response for estimating mean and compared the estimator under complete response. In this work we have explored the properties of proposed estimators for the following two cases, firstly when existence of non-response is considered on study variable and secondly when existence of non-response is considered on study variable and on auxiliary variable.

Let us consider a finite population whose size is N. We stratify the population into L strata which are homogenous in nature. N_h is considered as the size of i^{th} stratum $(i = 1,2,3, \dots, L)$ and $\sum_{i=1}^{L} N_i = N$. We select a sample of size n from the population in such a way that in every i^(th) stratum, we select a sample of size n_i from the N_i, where $\sum_{i=1}^{L} n_i = n$. Here Y and X are the study and auxiliary variable with population means $\bar{Y}$ and $\bar{X}$ respectively.

In this article following two cases are considered:

(i) When existence of non-response is considered on Y.

(ii) When existence of non-response is considered on Y and X both.

We have taken into account the following notations for their further use:

$\Re$: Set of Real numbers.

$\bar{y}_{st}$ and $\bar{x}_{st}$ are the sample means of Y and X respectively.

$\bar{y}_{ni1}$: Mean for the i^{th} stratum based on n_{i1} units of response group in the sample.

$\bar{y}_{ui2}$: Mean for the i^{th} stratum based on u_{i2} units of non-response group in the sample.

$\bar{y}_{st}^{*}$: Hansen-Hurwitz (1946) unbiased estimator for population mean $\bar{Y}$ under stratified sampling.

S_{Yi}^2,S_{Yi2}^2 : Mean square of entire group of study variable (including response and non-response group) and non-response group of study variable in the population for the i^{th} stratum respectively.

$W_{i2} = \frac{N_{i2}}{N_i}$: Non-response rate in the i^{th} startum of the population.

$m_i = \frac{n_{i2}}{u_{i2}}$,$p_i = \frac{N_i}{N}$ and $f_i = \frac{N_i - n_i}{N_i n_i}$

ρ_i: Correlation coefficient in the i^{th} stratum between variable Y and X.

For these two cases we define following ratio type estimator for estimating the population mean $\bar{Y}$ in the existence of non-response.

2. Proposed Estimators

Here we define ratio type estimator for two cases first one when existence of non-response is taken only in auxiliary variable X and another one when existence of non-response is taken in study variable and auxiliary variable both

2.1 Ratio type estimator when existence of non-response is taken in variable X

We define following ratio type estimator when existence of non-response is taken only in variable X:

$$V_{Rk}^{*} = \bar{y}_{st}^{*} \left(\frac{\bar{X}}{\bar{x}_{st}}\right)^{k} \text{, where k=1,2 and 3} \quad (1)$$

Using this proposition in equation (1), we put forward a class of estimators V^{*}:

$$V^{*} = \sum_{k=1}^{3} \alpha_k V_{Rk}^{*} \qquad (\in T_1) \tag{2}$$

where T_1 is the set of all the feasible ratio type estimators for estimation of population mean $\bar{Y}$ when existence of non-response is taken only in study variable.
$\alpha_k^{'s}$ are the suitable chosen constants that are used for reducing the bias in the set of real numbers such that

$$\sum_{k=1}^{3} \alpha_k = 1 \qquad (\alpha_k \in \Re) \tag{3}$$

$\Re$ is the set of real numbers.

For attaining the Bias and the MSE expression of the suggested estimator V^{*} we have taken into account the following transformation:

$$\bar{y}_{st}^{*} = \bar{Y}(1+e_0^{*}),\ \bar{x}_{st}^{*} = \bar{X}(1+e_1^{*}) \text{ and } \bar{x}_{st} = \bar{X}(1+e_1) \tag{4}$$

such that $E(e_0^{*}) = E(e_1^{*}) = E(e_1) = 0$

Further following lemmas have been used:

Lemma 1: $E({e_0^{*}}^2) = \frac{V(\bar{y}_{st}^{*})}{\bar{Y}^2} = \frac{1}{\bar{Y}^2}\sum_{i=1}^{L} p_i^2 \left[f_i S_{Yi}^2 + \frac{(m_i-1)}{n_i} w_{i2} S_{Yi2}^2\right]$ (5)

Lemma 2: $E(e_1^2) = \frac{V(\bar{x}_{st})}{\bar{X}^2} = \frac{1}{\bar{X}^2}\sum_{i=1}^{L} p_i^2\, f_i S_{xi}^2$ (6)

Lemma 3: $E(e_0^{*}e_1) = \frac{\mathrm{cov}(\bar{y}_{st}^{*}, \bar{x}_{st})}{\bar{Y}\bar{X}} = \frac{1}{\bar{Y}\bar{X}}\sum_{i=1}^{L} p_i^2\, f_i \rho_i S_{Yi} S_{Xi}$ (7)

Lemma 4: $E({e_1^{*}}^2) = \frac{V(\bar{x}_{st}^{*})}{\bar{X}^2} = \frac{1}{\bar{X}^2}\sum_{i=1}^{L} p_i^2 \left[f_i S_{xi}^2 + \frac{(m_i-1)}{n_i} w_{i2} S_{Xi2}^2\right]$ (8)

Lemma 5: $E(e_0^{*}e_1^{*}) = \frac{Cov(\bar{y}_{st}^{*}, \bar{x}_{st}^{*})}{\bar{X}\bar{Y}} = \frac{1}{\bar{X}\bar{Y}}\sum_{i=1}^{L} p_i^2 \left[f_i S_{YiXi}^2 + \frac{(m_i-1)}{n_i} w_{i2} S_{XiYi2}^2\right]$ (9)

where $f_i = \frac{N_i - n_i}{N_i n_i}$

Using the transformations defined in equation (4) our proposed estimator V^{*} takes the following form

$$V^{*} = \sum_{k=1}^{3} \alpha_k \bar{y}_{st}^{*} \left(\frac{\bar{X}}{\bar{x}_{st}}\right)^{k} = \bar{Y}(1+e_0^{*}) \sum_{k=1}^{3} \alpha_k (1+e_1)^{-k} \tag{10}$$

Theorem 1: The Bias and the MSE expression of the estimator V^{*} to the first order of approximation are

$$B(V^{*}) = \frac{1}{\bar{Y}}\left[\sum_{k=1}^{3} \frac{k\alpha_k}{2} \{\sum_{i=1}^{L} f_i p_i^2 ((1+k)R^2 S_{Xi}^2 - 2R\rho_i S_{Yi} S_{Xi})\}\right] \tag{11}$$

$$MSE(V^*) = \left[\sum_{i=1}^{L} p_i^2 \frac{(m_i-1)}{n_i} w_{i2} S_{Yi2} + \sum_{i=1}^{L} f_i p_i^2 \{S_{Yi}^2 + A^2 R^2 S_{Xi}^2 - 2AR\rho_i S_{Yi} S_{Xi}\}\right] \quad (12)$$

where $A = \sum_{k=1}^{3} k\beta_k$

Proof: To attain the Bias of the estimator V^*, we expand the equation (10) and ignore the higher order terms,

$$V^* = \sum_{k=1}^{3} \alpha_k \bar{Y} \left[1 + e_0^* - ke_1 + \frac{k(k+1)}{2} e_1^2 - ke_0^* e_1\right] \quad (13)$$

We subtract $\bar{Y}$ from both the sides of equation (13) and took expectation of the equation (13), we attain our expression of bias as

$$B(V^*) = E[V^* - \bar{Y}] = \frac{1}{\bar{Y}}\left[\sum_{k=1}^{3} \frac{k\alpha_k}{2} \{\sum_{i=1}^{L} f_i p_i^2 ((k+1)R^2 S_{Xi}^2 - 2R\rho_i S_{Yi} S_{Xi})\}\right] \quad (14)$$

MSE expression for the estimator V^* is

$$MSE(V^*) = E[V^* - \bar{Y}]^2 = \bar{Y}^2 E[\sum_{k=1}^{3} \alpha_k e_0^* - \sum_{k=1}^{3} k\alpha_k e_1]^2 \quad (15)$$

Expanding RHS of equation (15) and taking expectation, we attain $MSE(V^*)$ as given in equation (12)., where $A = \sum_{k=1}^{3} k\alpha_k$

2.1.1 Almost unbiased version of the estimator V^*

Now differentiate equation (12) with respect to A and equate the result to zero, we attain the minimum $MSE(V^*)$ at

$$A = \frac{\sum_{i=1}^{L} f_i p_i^2 \rho_i S_{Yi} S_{Xi}}{R \sum_{i=1}^{L} f_i p_i^2 S_{Xi}^2} \quad (16)$$

Putting the value of A in MSE expression (12), we attain the minimum MSE of the estimator V^* as

$$Min. MSE(V^*) = \sum_{i=1}^{L} p_i^2 \frac{(m_i-1)}{n_i} w_{i2} S_{Yi2}^2 + \sum_{i=1}^{L} f_i p_i^2 S_{Yi}^2 - \frac{(\sum_{i=1}^{L} f_i p_i^2 \rho_i S_{Yi} S_{Xi})^2}{\sum_{i=1}^{L} f_i p_i^2 S_{Xi}^2} \quad (17)$$

By applying following intrinsic conditions, the propounded classes of estimators are made almost unbiased.

(i) The sum of weights is one i.e., $\alpha_1 + \alpha_2 + \alpha_3 = 1$

(ii) The approximate biases of the propounded classes of estimators are zero i.e. $B(V^*) = 0$

(iii) MSE of estimator attain minimum under optimum condition.

For achieving the third condition MSE has been minimized with respect to A which gives the solution of A as:

$$A = \sum_{k=1}^{3} k\beta_k = \frac{\sum_{i=1}^{L} f_i p_i^2 \rho_i S_{Yi} S_{Xi}}{R \sum_{i=1}^{L} f_i p_i^2 S_{Xi}^2} = A_1(say) \quad (18)$$

From the above three conditions we get three simultaneous equations to determine the unique values of scalar $\alpha_k^{'s}$ (k = 1,2,3).

$$\alpha_1 + \alpha_2 + \alpha_3 = 1 \qquad (19)$$

$$\alpha_1(1 - A_1) + \alpha_2(3 - 2A_1) + 3\alpha_3(2 - A_1) = 0 \qquad (20)$$

$$\alpha_1 + 2\alpha_2 + 3\alpha_3 = A_1 \qquad (21)$$

Solving the equation (19), (20) and (21) we get unique values of scalar $\alpha_k^{'s}$ (k = 1,2,3) as follows:

$$\alpha_1 = {A_1}^2 - 3A_1 + 3 \qquad (22)$$

$$\alpha_2 = -2{A_1}^2 + 5A_1 - 3 \qquad (23)$$

$$\alpha_3 = {A_1}^2 - 2A_1 + 1 \qquad (24)$$

Substituting the values of $\alpha_k^{'s}$ in equation (12) we attain an AUORE V_{OE}^* of the population mean as:

$$V_{OE}^* = \left[({A_1}^2 - 3A_1 + 3)\bar{y}_{st}^*\left(\frac{\bar{X}}{\bar{x}_{st}}\right) + (-2{A_1}^2 + 5A_1 - 3)\bar{y}_{st}^*\left(\frac{\bar{X}}{\bar{x}_{st}^*}\right) + ({A_1}^2 - 2A_1 + 1)\bar{y}_{st}^*\left(\frac{\bar{X}}{\bar{x}_{st}}\right)\right] \qquad (25)$$

where $A_1 = \frac{\sum_{i=1}^{L} f_i p_i^2 \rho_i S_{Yi} S_{Xi}}{R\sum_{i=1}^{L} f_i p_i^2 S_{Xi}^2}$.

The variance of AOURE V_{OE}^* of population mean is

$$Var(V_{OE}^*) = \sum_{i=1}^{L} p_i^2 \frac{(m_i-1)}{n_i} w_{i2} S_{Yi2}^2 + \sum_{i=1}^{L} f_i p_i^2 S_{Yi}^2 - \frac{(\sum_{i=1}^{L} f_i p_i^2 \rho_i S_{Yi} S_{Xi})^2}{\sum_{i=1}^{L} f_i p_i^2 S_{Xi}^2} \qquad (26)$$

2.2 Ratio type estimator when existence of non-response is taken in study variable Y and auxiliary variable X both

We define second class of ratio type estimator when existence of non-response is considered in study variable Y and auxiliary variable X both.

$$V_{Ri}^{**} = \bar{y}_{st}^*\left(\frac{\bar{X}}{\bar{x}_{st}^*}\right)^k \qquad (27)$$

where k=1,2 and 3

Using the above preposition in equation (27) we put forward the following estimator:

$$V^{**} = \sum_{k=1}^{3} \beta_k V_{Rk}^{**} \quad (\in T_2), \qquad (28)$$

where T_2 is the set of all feasible ratio type estimators for estimating population mean $\overline{Y}$ when non-response is considered in Y as well as X.

β_k denotes suitably chosen constants used for reducing bias in the set of real numbers such that

$$\sum_{k=1}^{3} \beta_k = 1 \quad (\beta_k \in \Re) \tag{29}$$

Similarly, as in previous case, here we have obtained bias and MSE expression and applied similar intrinsic conditions as in the previous one and the class of estimators V^{**} is made almost unbiased.

Using approximations:

Theorem 2: Bias and the MSE expression of the estimator V^{**} up to the first order of approximation are as follows:

$$B(V^{**}) = \frac{1}{\bar{Y}}\left[\sum_{k=1}^{3} \frac{k\beta_k}{2}\left\{\sum_{i=1}^{L} f_i p_i^2 [(k+1)R^2 S_{Xi}^2 - 2RS_{YiXi}] + \sum_{i=1}^{L} \frac{(m_i-1)}{n_i} p_i^2 w_{i2} S_{Xi2}^2 - 2S_{XiYi2}^2\right\}\right] \quad \ldots (30)$$

$$MSE(V^{**}) = \sum_{i=1}^{L} p_i^2 f_i (S_{Yi}^2 + B^2 R^2 S_{Xi}^2 - 2BRS_{XiYi}^2) + \sum_{i=1}^{L} p_i^2 \frac{(m_i-1)}{n_i} w_{i2} (S_{Yi2}^2 + B^2 R^2 S_{Xi2}^2 - 2BRS_{XiYi2}^2) \quad \ldots (31)$$

where $B = \sum_{k=1}^{3} k\beta_k$

Proof: expand equation (28) in terms of e_0 and e_1

$$V^{**} = \sum_{k=1}^{3} \beta_k \bar{Y}\left(1 + e_0^* - ke_1^* + \frac{k(1+k)}{2} e_1^{*^2} - ke_1^* e_0^*\right) \tag{32}$$

Subtract $\bar{Y}$ from both the sides of equation (32) and take expectation to get the bias of V^{**} as

$$B(V^{**}) = E[V^{**} - \bar{Y}] = \bar{Y} \sum_{k=1}^{3} \frac{k\beta_k}{2}\left[(k+1)E\left(e_1^{*^2}\right) - E(e_1^* e_0^*)\right] \tag{33}$$

And proceeding in the similar manner as in previous case we get expressions for bias as well as MSE given by equation (30) and (31) simultaneously.

2.2.1 Almost unbiased version of the estimator V^*

To get the optimum value of B, we differentiate equation (31) with respect to B and equate it to zero.

$$B = \frac{\sum_{i=1}^{L} p_i^2 f_i S_{YiXi}^2 + \sum_{i=1}^{L} p_i^2 \left(\frac{m_i-1}{n_i}\right) w_{i2} S_{XiYi2}}{R\sum_{i=1}^{L} p_i^2 f_i S_{Xi}^2 + \sum_{i=1}^{L} p_i^2 \frac{(m_i-1)}{n_i} w_{i2} S_{Xi2}} = B_1(say) \tag{34}$$

where $R = \frac{\bar{Y}}{\bar{X}}$

$$Min.MSE(V^{**}) = \sum_{i=1}^{L} p_i^2 \left(f_i S_{Yi}^2 + \frac{(m_i-1)}{n_i} w_{i2} S_{Yi2}^2\right) - \frac{\left[\sum_{i=1}^{L} p_i^2 \left(f_i S_{YiXi} + \frac{(m_i-1)}{n_i} w_{i2} S_{XiYi2}\right)\right]^2}{\sum_{i=1}^{L} p_i^2 \left[f_i S_{Xi}^2 + \frac{(m_i-1)}{n_i} w_{i2} S_{Xi2}^2\right]} \tag{35}$$

Proceeding similarly as before we get optimum values of $\beta_1, \beta_2, \beta_3$ as

$$\left.\begin{aligned} \beta_1 &= B_1^2 - 3B_1 + 3 \\ \beta_2 &= -2B_1^2 + 5B_1 - 3 \\ \beta_3 &= B_1^2 - 2B_1 + 1 \end{aligned}\right\} \tag{36}$$

Substituting these values of $\beta_k^{'s}$ $(k = 1,2,3)$ from equation (36) in equation (28) we get AUORE of population mean as

$$V_{OE}^{**} = \left[(B_1^2 - 3B_1 + 3)\bar{y}_{st}^*\left(\frac{\bar{X}}{\bar{x}_{st}^*}\right)^1 + (-2B_1^2 + 5B_1 - 3)\bar{y}_{st}^*\left(\frac{\bar{X}}{\bar{x}_{st}^*}\right)^2 + (B_1^2 - 2B_1 + 1)\bar{y}_{st}^*\left(\frac{\bar{X}}{\bar{x}_{st}^*}\right)^3\right] \quad \text{... (37)}$$

Where $B_1 = \dfrac{\sum_{i=1}^{L} p_i^2 f_i S_{YiXi}^2 + \sum_{i=1}^{L} p_i^2 \left(\frac{m_i - 1}{n_i}\right) w_{i2} S_{XiYi2}}{R\sum_{i=1}^{L} p_i^2 f_i S_{Xi}^2 + \sum_{i=1}^{L} p_i^2 \frac{(m_i-1)}{n_i} w_{i2} S_{Xi2}}$

And the variance of the AUORE V_{OE}^{**} to the first degree of approximation is obtained as

$$Var(V_{OE}^{**}) = \sum_{i=1}^{L} p_i^2\left(f_i S_{Yi}^2 + \frac{(m_i - 1)}{n_i} w_{i2} S_{Yi2}^2\right) - \frac{\left[\sum_{i=1}^{L} p_i^2\left(f_i S_{YiXi} + \frac{(m_i-1)}{n_i} w_{i2} S_{XiYi2}\right)\right]^2}{\sum_{i=1}^{L} p_i^2\left[f_i S_{Xi}^2 + \frac{(m_i - 1)}{n_i} w_{i2} S_{Xi2}^2\right]} \quad \text{... (38)}$$

3. Comparison with $\bar{y}_{st}^*$ and natural ratio estimator under non-response

Natural ratio estimators V_{R1} and V_{R2} in existence of non-response defined for two circumstances are:

$V_{R1} = \bar{y}_{st}^*\left(\frac{\bar{X}}{\bar{x}_{st}}\right)$; when existence of non-response is taken only in Y.

$V_{R2} = \bar{y}_{st}^*\left(\frac{\bar{X}}{\bar{x}_{st}^*}\right)$; when existence of non-response is taken in Y as well as X

The MSE of estimators Y_{R1} *and* V_{R2} are given by

$$MSE(V_{R1}) = \left[\sum_{i=1}^{L} p_i^2 \frac{(m_i-1)}{n_i} w_{i2} S_{Yi2}^2 + \sum_{i=1}^{L} f_i p_i^2 (S_{Yi}^2 + R^2 S_{Xi}^2 - 2R\rho_i S_{Yi} S_{Xi})\right] \quad (39)$$

$$MSE(V_{R2}) = \left[\sum_{i=1}^{L} p_i^2 \frac{(m_i-1)}{n_i} w_{i2}(S_{Yi2}^2 + R^2 S_{Xi2}^2 - 2RS_{XiYi}) + \sum_{i=1}^{L} p_i^2 f_i (S_{Yi}^2 + S_{Xi}^2 - 2RS_{XiYi})\right] \quad \text{...(40)}$$

Since $\vec{y}_{st}^*$ is an unbiased estimator of $\bar{Y}$, therefore variance of $\vec{y}_{st}^*$ is given by

$$Var(\bar{y}_{st}^*) = \sum_{i=1}^{L}\left(\frac{1}{n} - \frac{1}{N}\right) p_i^2 S_{Yi}^2 + \sum_{i=1}^{L} \frac{(m_i-1)}{n_i} w_{i2} p_i^2 S_{Yi2}^2 \quad (41)$$

Percent relative efficiency (PRE) of the proposed estimators with respect to usual estimator $\vec{y}_{st}^*$ and natural estimator V_{R1} and V_{R2}.

$$PRE(V_{OE}^*) = P_1 = \frac{MSE(\bar{y}_{st}^*)}{Var(V_{OE}^*)} \times 100 \quad (42)$$

$$PRE(V_{OE}^{**}) = P_2 = \frac{MSE(\bar{y}_{st}^*)}{Var(V_{OE}^{**})} \times 100 \quad (43)$$

$$PRE(V_{OE}^*) = P_3 = \frac{MSE(V_{R1})}{Var(V_{OE}^*)} \times 100 \quad (44)$$

$$PRE(V_{OE}^{**}) = P_4 = \frac{MSE(V_{R2})}{Var(V_{OE}^{**})} \times 100 \quad (45)$$

4. Comparison with estimator under complete response

Estimator under complete response is given as

$$V_{OE} = \sum_{k=1}^{3} \beta_k \bar{y}_{st} \left(\frac{\bar{X}}{\bar{x}_{st}} \right)^k \quad (46)$$

The variance of the estimator V_{OE} is given by

$$Var(V_{OE}) = \sum_{i=1}^{k} f_i\, p_i^2 (S_{Yi}^2 + R^2 S_{Xi}^2 - 2R\rho_i S_{Yi} S_{Xi}) \quad (47)$$

5. Numerical analysis

For numerical analysis we have considered the following data set.

Population sources (I): Sarjinder Singh (2003) Page No. 762 and obtained the following results:

In a circus there are three types of elephants, viz., light, medium, and heavy in weight and some information about them is listed below in table 1:

x: Food, kg/day

y: weight, kg

Table 1. Stratum means, Mean square errors and correlation coefficients

Stratum No.	N_i	n_i	Y_i	X_i	S_{Yi}	S_{Xi}	S_{YiXi}	ρ_i
1	250	20	2000	100	500	60	17500	0.58
2	150	12	3500	150	400	30	8400	0.70
3	100	8	5000	150	200	20	2800	0.70

Optimum values of scalars $\alpha_j^{'s}$ and $\beta_j^{'s}$ (j=1,2 and 3), presented in table 2 and table 3, used in the estimators reduces bias to the first order of approximation.

Table 2. optimum values of $\alpha_j^{'s}$ (j = 1,2 and 3)

Scalars	Population (I)
α_1	1.9442
α_2	-1.2956
α_3	0.3514

Table 3. optimum values of $\beta_j^{'s}$ (j = 1,2 and 3) at $w_{i2} = 0.1$ and $k_i = 2$

Scalars	Population (I)
β_1	2.2681
β_2	-1.8040

β_3	0.5359

Table 4. PRE table for population I

w_{i2}	m_{i2}	$PRE(V_{OE}^{*})$ with respect to $\bar{y}_{st}^{*}$	$PRE(V_{OE}^{**})$ with respect to $\bar{y}_{st}^{*}$	$PRE(V_{OE}^{*})$ with respect to V_{R1}	$PRE(V_{OE}^{**})$ with respect to V_{R2}
0.1	2	155.12	160.51	347.45	240.37
	2.5	154.61	162.78	345.98	285.29
	3	154.12	165.11	344.53	330.99
	3.5	153.63	167.51	343.09	377.52
0.2	2	154.12	165.11	344.53	330.99
	2.5	153.15	169.97	341.68	424.92
	3	152.21	175.12	338.89	522.52
	3.5	151.32	180.57	336.17	624.16
0.3	2	153.15	169.97	341.68	424.92
	2.5	151.76	177.80	337.52	572.81
	3	150.45	186.35	333.51	730.27
	3.5	149.20	195.70	329.63	898.83

As given in the table 4, PRE of our suggested estimators V_{OE}^{*} and V_{OE}^{**} with respect to existing estimators shows that proposed estimators performs better than the $\bar{y}^{*}{}_{st}$ and natural ratio estimator.

6. Conclusion

We have proposed class of AUORE in stratified sampling using auxiliary information by considering the existence of non-response on (i) study variable only, (ii) study variable as well as auxiliary variable. Then we have derived the expression for the bias and the min MSE in each case. From the table 4 we come to an end that the suggested estimators perform better than $\bar{y}_{st}^{*}$ estimator and natural ratio estimator under non-response, and therefore our presented class of estimators is suggested for practical application under both the discussed circumstances.

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Conflicts of Interest

The authors declare no conflict of interest.

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