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ARTICLE

The effects of inflation, shortages, and partial backlogs on products that deteriorate over time in response to varied demand

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Abstract
This study aims to determine an optimal policy to deal with a situation where a retailer should have enough stock to meet customer demands and prevent supply loss and deterioration. In this study, we demonstrate the impact of partial backlogging, shortages, and inflation on decaying items to provide the most relevant solutions to this problem. The model uses a two-parameter Weibull deterioration distribution with an exponential demand rate. Numerical results and sensitivity analysis are combined with a graphical demonstration, and the proposed approach is reliable and accurate, providing a new explanation for a different type of system. This paper determines the total cost and optimal production run time by developing an inventory model. Sensitivity analysis is carried out to demonstrate the proposed model.

Keywords: Inflation; Weibull deterioration; Shortages; Stock Varying Demand; Partial backlogging

1. Introduction

Decay refers to the change, rotting, destruction, damage, spoilage, evaporation, degradation, etc., in the marginal value of a product or utility that decreases the value of the original or fresh product in stock. Seafood, medicine, blood, fruits, gasoline, vegetables, clothes, machinery, chemicals, and other products have a finite shelf-life cycle, and deterioration begins immediately after replenishment. In addition, deterioration is typically observed in some commodities during the average storage period.

Maximum inventory models consider that the demand is either backlogged in the period of no stock or that all demand is lost, which is unrealistic. In the actual situation, some customers want to wait until replenishment, mainly if the waiting period is short or because of certain regions, such as personal relationships. It is also observed that some products, such as medicines and vegetables, are required instantly to fulfil their requirements; they must go elsewhere to satisfy their demand. The backlogging rate is also critical in replenishment and depends on the time. The longer the customers wait, the more significant the fraction of lost sales. Most researchers working on the same concept must simultaneously consider the facts, such as demand, deterioration rate, inflation, and shortage.

To provide the most relevant solution for these problems, a two-parameter Weibull deterioration distribution is considered with an exponential demand, increasing with time. Shortages are allowed which are partially backlogged. Furthermore, we consider inflation. In this study, we determine an optimal policy to deal with a situation where a retailer should have sufficient stock to meet customer demands and prevent stock-out and deterioration. This study also demonstrates the impacts of partial backlogging, shortages, and inflation on decaying items.

The numerical results and sensitivity analysis are combined with a graphical demonstration. A mathematical formation of the model is presented to support the theory, and sensitivity analysis is carried out to validate the model.

2. Literature Review

It is always observed that there is a shortage of inventory in case of an emergency or calamity. At that time, fulfilling demand for every consumer becomes difficult as inventory storage problems arise. Therefore, warehouses can be used to store inventories during emergencies. The use of warehouses is an essential and beneficial way to maintain inventory. The warehouse inventory issue has received considerable attention in recent years. Many researchers have developed several models using one or more warehouses. In a multi-warehouse system, it is assumed that the merchant owns the warehouse (OW) with an unchanging capability and a quantity that should be stocked in the borrowed warehouse. Chung, K.-J., & Huang, T. S. (2007) discussed the inventory models of two warehouses for deteriorating items; on the other hand, Singh, S. R. and Bhatia, D. (2011) proposed two warehouse models through inflation-induced demand, whereas Chang, J. and Lin, F. (2010) discussed the improved model considering optimal replenishment policy under the demand price dependent on stock. Recently, Sindhuja and Aarthi (2023) elaborated an inventory policy for perishable commodities and used the preservation technology with time-based demand. Some researchers believe in a warehouse with unlimited capacity because it can be challenging to select which type of warehouse should be filled or vacant first in the case of owned and rented warehouses. To overcome this problem, we are considering a warehouse with unlimited capacity.

Khanra, S., Ghosh, S. K. and Chaudhuri, K. S. (2011) discussed an inventory model for decaying items with quadratic demand under the permissible delay payments condition and similar work also done by Liang, Y., & Zhou, F. (2011) considered a concept identical to that of a two-warehouse for the permissible delay in payment. Recently, Handa, N., Singh, S. R., and Punetha, N. (2021) investigated a production policy that considered ramp-type demand for the inventory system with shortages and inflation. Goel, A. and Ali, M. A. (2022) considered a model for incremental holding costs with the effect of stock-dependent deterioration, including partial backlogging. Samih Antoine Azar (2023) discussed a scheme without backorders and non-deterministic demand.

Singh, S. R., Gupta, V. and Goel, A. (2013) consider an EOQ model with a trade credit policy for items that deteriorate with time, and the demand changes with the change in the selling price; Kumar, M., Chauhan, A., Singh, S. J. and Sahni, M. (2020) recently extended the above research on trade credit with preservation technology under demand depending on advertisement, time, and selling price. Singh, A. and Goel, A. (2024) developed a mathematical framework for managing decaying products in multiple warehouses within a retail setting under a trade credit policy. Atama. A, Madaki and Sani. B. A (2024) suggested a manufacturing inventory policy with linear time-related demand and

constant holding cost. They allowed shortages in their study.
Yadav, A. S. and Swami, A. (2018) discussed the inventory lot size model for partially backlogging with the holding cost as time-varying and Weibull deterioration. Singh, S. R. and Rana, K. (2020) discussed and proposed a mathematical model considering the optimal refill policy for the new product and take-return of used product quantity for deteriorating items with the concept of lead time. Goel, A. & Singh, A. (2024) examined a model for perishable commodities with different degradation rates. To slow down a product's rate of deterioration, Shah, N. H., Jani, M. Y., and Chaudhari (2018) considered preservation investment. Arora, R., Singh, A. P., Sharma, R., & Chauhan, A. (2021) created a traditional EOQ model with shortage considering a fuzzy environment and provided an appropriate structure to handle such uncertain parameters. Recently, Mou, J. J., & Jiang, Y. M. (2021) discussed an integrated cold chain policy for the Weibull deterioration rate based on inventory performance. Md. Al-Amin Khan, Leopoldo, Eduardo. (2024) analyzed the effects of flexible advance payments on cost and inventory decisions with an energy demand model and nonlinear holding costs under carbon cap and price regulation.

After a critical review, it is observed that realistic scenarios, such as shortages, partial backlogging, time-dependent deteriorating items, and the effect of inflation, are not considered simultaneously by most researchers.

Therefore, in the present study, we incorporate all the above-discussed scenarios, consider shortages, and obtain the optimal ordering policy, which deals with the situation in which a retailer should have enough stock to meet the customer's demands to prevent stock-out and deterioration.

3. Assumptions and Notations

- The demand pattern is assumed to be time-dependent.
- The production rate is dependent on the demand of the product i.e., $\boldsymbol{P}(\boldsymbol{t}) = \boldsymbol{\alpha D}(\boldsymbol{t})$.
- The rate of product deterioration is the function of time "t".
- The role of inflation was also considered.
- The partial backlogging of shortages is admissible.
- The warehouse has limitless capacity.
- Deteriorating products are rejected entirely.

$\boldsymbol{I}(\boldsymbol{t})$	The stock level at a certain moment "t".
$\boldsymbol{A}$	The set-up cost per cycle.
$\boldsymbol{h}$	Per unit time holding cost.
$\boldsymbol{\theta}(\boldsymbol{t})$	The deterioration rate is the Weibull distribution of two parameters where $\boldsymbol{\theta}(\boldsymbol{t}) = \boldsymbol{\beta\gamma t^{\gamma-1}\beta}(\mathbf{0} < \boldsymbol{\beta} < \mathbf{1})$ is the scale parameter and it is a probability density function and$\boldsymbol{\gamma}(\boldsymbol{\gamma} > \mathbf{0})$ is the shape parameter.
$\boldsymbol{D}(\boldsymbol{t})$	The demand rate is an exponentially increasing function of time$\boldsymbol{D}(\boldsymbol{t}) = \boldsymbol{a}\ \boldsymbol{e^{bt}}$, where "$a$" and "b" are the demand parameters.
$\boldsymbol{P}(\boldsymbol{t})$	The manufacturing rate depends on the demand rate, i.e.$\boldsymbol{P}(\boldsymbol{t}) = \boldsymbol{\alpha}\ \boldsymbol{ae^{bt}}$ where $\boldsymbol{\alpha} > \mathbf{1}$
$\boldsymbol{S}$	Maximum stock level.
$\boldsymbol{T}$	Cycle length
$\boldsymbol{v}$	Stock level when time becomes zero.
$\boldsymbol{t_1}$	The time up to which production occurs.
$\boldsymbol{d}$	Per unit time Deterioration cost.
$\boldsymbol{p}$	The production cost per unit of time.
$\boldsymbol{s}$	Per unit time shortages cost.
$\boldsymbol{l}$	The lost sales per unit time.
$\boldsymbol{\delta}$	The rate of backlogging.
$\boldsymbol{r}$	The rate of inflation.

M	Retailers trade credit period, which the supplier provides.
$\boldsymbol{i_e}$	Interest earned by retailer.
$\boldsymbol{i_P}$	Interest paid by the retailer

4. Mathematical Modelling

In this model, the producer starts production at the time $t = 0$, and continues until the time after satisfying the demand for deterioration during the complete cycle. There is a decline in the inventory level, which becomes zero because of the outcome of the demand and deterioration only before the shortage occurs. The suggested model shows variations in inventory level over an assumed period while also considering the impact of inflation. $t = t_1[t_1, v]$.

The basic process is explained with the help of the following figure 1.

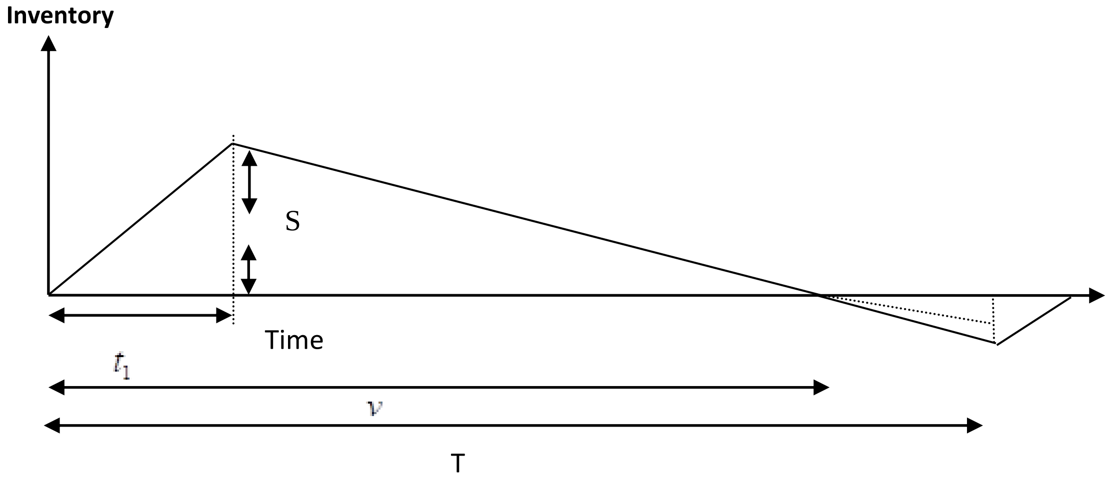


Figure 1. Representation of different levels of inventory.

Differential equations always show a change in one variable or parameter for another variable or parameter**,** so the differential equations that described the above system are:

$$\frac{dI_1(t)}{dt} = P(t) - [D(t) + \theta(t)I_1(t)] \qquad 0 \le t \le t_1 \qquad (1)$$

$$\frac{dI_1(t)}{dt} + \beta\gamma t^{\gamma-1} I_1(t) = ae^{bt}(\alpha - 1)$$

$$\frac{dI_2(t)}{dt} + \theta(t)I_2(t) = -D(t) \qquad t_1 \le t \le v \qquad (2)$$

$$\frac{dI_2(t)}{dt} + \beta\gamma t^{\gamma-1} I_2(t) = -ae^{bt}$$

Boundary conditions for the above equations concerning the considered model are given below.

$$I_1(0) = 0, I_2(t_1) = S \qquad (3)$$

Solutions for the above equations considering boundary values are as follows:

$$\boldsymbol{I_1(t) = a(\alpha - 1)e^{-\beta t^\gamma}\left(\frac{\beta}{\gamma+1}t^{\gamma+1} + \frac{b}{2}t^2 + t\right) 0 \le t \le t_1} \qquad (4)$$

$$I_2(t) = \left\{Se^{\beta t_1^\gamma} + a\left((t_1 - t) + \frac{b}{2}(t_1^2 - t^2) + \frac{\beta}{\gamma+1}(t_1^{\gamma+1} - t^{\gamma+1})\right)\right\} e^{-\beta t^\gamma}, t_1 \le t \le v \qquad (5)$$

If $I_2(v) = 0$.

From equation (5), We have

$$S = a\left\{(v - t_1) + \frac{b}{2}(v^2 - t_1^2) + \frac{\beta}{\gamma+1}(v^{\gamma+1} - t_1^{\gamma+1})\right\} e^{-\beta t_1^\gamma} \qquad (6)$$

5. Analysis of Different Costs

5.1 Set-Up Cost:

The setup cost associated with this model is displayed as follows:

$$\text{Setup Cost} = A \tag{7}$$

5.2 Carrying or Holding Cost:

The holding cost *(H.C.)* involved in this model is given as:

$$H.C. = h\left\{\int_0^{t_1} e^{-rt}.I_1(t)dt + \int_{t_1}^{v} I_2(t)e^{-rt}dt\right\} \tag{8}$$

(See appendix (a))

5.3 Deterioration Cost:

The deterioration cost (*D.C.)* associated with this model is as follows:

Deteriorated units = d (Total production cost - total det. cost)

$$= d\left\{\int_0^{t_1} \alpha a e^{bt} e^{-rt}\, dt - \int_0^{v} a e^{bt} e^{-rt} dt\right\}$$

$$D.C. = d\left\{\int_0^{t_1} \alpha a e^{(b-r)t}\, dt - \int_0^{v} a e^{(b-r)t} dt\right\}$$

$$\boldsymbol{D.C. = d\left\{a(\alpha t_1 - v) + a(b-r)\left(\frac{\alpha t_1^2}{2} - \frac{v^2}{2}\right)\right\}} \tag{9}$$

5.4 Production Cost:

The production cost *(P.C.)* associated with this model is given below:

Total produced unit= $p\int_0^{t_1} \alpha\, a e^{bt} e^{-rt} dt$

$$P.C. = a\alpha p\left(t_1 + (b-r)\frac{t_1^2}{2}\right) \tag{10}$$

5.5 Shortage Cost:

The shortage cost *(S.C.)* associated with this model is given by:

Total shortages= $\delta s\int_v^{T} a e^{bt} e^{-rt} dt$

$$S.C. = as\delta\left((T-v) + \frac{(b-r)}{2}(T^2 - v^2)\right) \tag{11}$$

5.6 Lost Sale Cost:

Lost sale cost *(L.S.C)* associated with this model is given by:

$$L.S.C = l.\int_v^{T} (1-\delta) a e^{bt} e^{-rt} dt$$

$$L.S.C = al(1-\delta)\left(T - v + \frac{(b-r)}{2}(T^2 - v^2)\right) \tag{12}$$

6. Cases of Permissible Delay in Payment:

Now for the case of permissible delay in payment, the following three cases arise:

6.1 When $\boldsymbol{0 \le M \le t_1}$:

In this case, interest payable = $\boldsymbol{IP_1 = pi_p\left[\int_M^{t_1} I_1(t)e^{-rt}dt + \int_{t_1}^{v} I_2(t)e^{-rt}dt\right]}$

(See appendix (b))

Interest earned = $\boldsymbol{IE_1 = pi_e\int_0^{M} D(t)te^{-rt}dt = \frac{pi_e a[3-2(b-r)]M^2}{6}}$

6.2 When $\boldsymbol{t_1 \le M \le v}$:

In this case, interest payable = $IP_2 = pi_p\left[\int_M^{v} I_2(t)e^{-rt}dt\right]$

(See appendix (c))

Interest earned $= IE_2 = pi_e \int_0^M D(t)te^{-rt}dt = \frac{pi_e a[3-2(b-r)]M^2}{6}$

6.3 When $T \leq M$:

In this case, there is no interest payable.

Interest earned is given as

$$IE_2 = pi_e \left[\int_0^M D(t)te^{-rt}d\ t + \frac{a(M-t_1)(e^{(b-r)t_1}-1)}{b-r}\right]$$
$$= \frac{pi_e a[3-2(b-r)]M^2}{6} + \frac{a(M-t_1)\left(e^{(b-r)t_1}-1\right)}{b-r}$$

7. Total Average Cost:

This model consists of the following total average cost (*T.A.C.*):

$$T.A.C. = \frac{1}{T}\begin{cases} TC_1(v,T)\ ,\ 0 \leq M \leq t_1 \\ TC_2(v,T)\ ,\ t_1 \leq M \leq v \\ TC_2(v,T)\ ,\ T \leq M \end{cases}$$

$T.C. =$ [Set up cost (S)+ production cost(P.C.) + inventory holding cost (I.H.C.) + deterioration cost (D.C.) + shortages cost (S.C.) + lost sale cost (L.S.C) + Interest payable -Interest earned]

$$TC_1 = S + P.C. + I.H.C. + D.C. + S.C. + L.S.C + IP_1 - IE_1$$
$$TC_2 = S + P.C. + I.H.C. + D.C. + S.C. + L.S.C + IP_2 - IE_2$$
$$TC_3 = S + P.C. + I.H.C. + D.C. + S.C. + L.S.C - IE_3$$

8. Numerical Example:

The model's applicability can be seen in the subsequent numerical example. The values of the involved parameters are taken as follows:

Table 1. Value of parameters

a	100 units
b	10 units
β	0.05
γ	2
l	3 Rs/unit
h	0.6 Rs/unit
d	8 Rs/unit
s	2 Rs/unit
p	4 Rs/unit
T	15
α	1.2
A	150 Rs/production run
r	0.05
δ	0.03
M	3500

Using the above-cited values, the optimal value for the production period (t_1) & critical point (v) carried the value 0.1253 days and 9.489 days, respectively.

Analysis is conducted to determine the Total Average Cost's ideal value which is Rs. 352.5180 $(T.A.C.)$

The following graphical representation is for the total average cost function, which shows its convex nature.

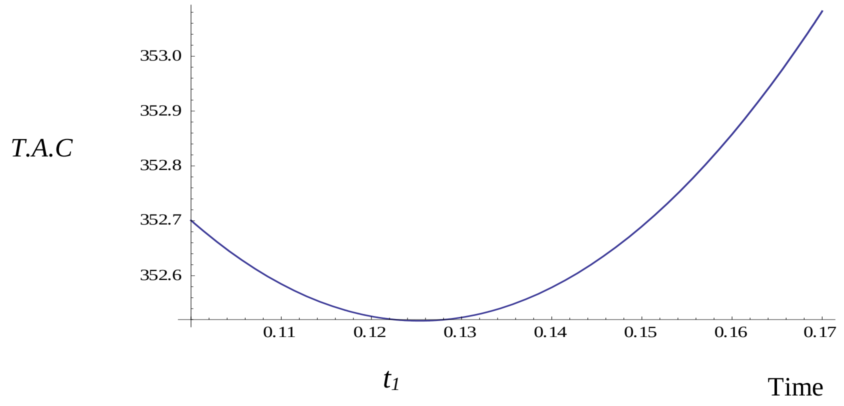


Figure 2. Convexity: total average cost function w.r.t. at$t_1 v = 9.4890$.

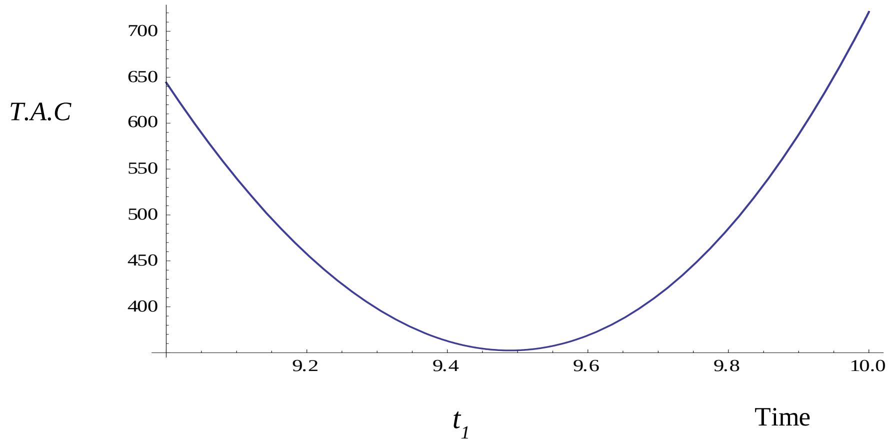


Figure 3. Convexity: total average cost function w.r.t. at v $t_1 = 0.1253$.

9. Sensitivity Analysis

The parameters' sensitivity is measured to check their impact on the total average cost. Tables 1 to 11 represent modification in TAC by considering one parameter at a time and keeping the remaining parameter unchanged.$(T.A.C.) a, b, \delta, \beta, h, d, s, m, l, \alpha, r$

Table 2. Sensitivity study of demand parameter $\boldsymbol{a}$

a	v	t_1	$T.A.C.$
80	9.5437	0.2019	80.0942
85	9.5275	0.1781	149.1460
90	9.5132	0.1579	217.4880
95	9.5004	0.1405	285.2470
100	9.4890	0.1253	352.5180
105	9.4786	0.1120	419.3780
110	9.4693	0.1002	485.8870
115	9.4608	0.0897	552.0960
120	9.4530	0.0802	618.0440

Table 3. Sensitivity study of demand parameter $\boldsymbol{b}$

b	v	t_1	$T.A.C.$
8.0	9.5106	0.1669	137.2180
8.5	9.5042	0.1541	191.2290
9.0	9.4986	0.1431	245.1000
9.5	9.4935	0.1336	298.8570
10.0	9.4890	0.1253	325.5180
10.5	9.4848	0.1180	406.0980
11.0	9.4811	0.1115	459.6090
11.5	9.4777	0.1057	513.0610
12.0	9.4745	0.1004	566.4620

Table 4. Sensitivity study of backlogging rate$\boldsymbol{\delta}$

δ	v	t_1	$T.A.C.$
0.0240	9.4904	0.1254	379.5920
0.0255	9.4901	0.1254	372.8240
0.0270	9.4897	0.1254	366.0560
0.0285	9.4893	0.1253	359.2870
0.0300	9.4890	0.1253	352.5180
0.0315	9.4886	0.1253	345.7480
0.0330	9.4882	0.1253	338.9790
0.0345	9.4879	0.1252	332.2080
0.0360	9.4875	0.1252	325.4380

Table 5. Sensitivity study of deterioration parameter$\boldsymbol{\beta}$

β	v	t_1	$T.A.C.$
0.0450	9.82105	0.119584	-1232.3900
0.0475	9.64926	0.122456	-404.9400
0.0500	9.48901	0.125362	352.5180
0.0525	9.33904	0.12831	1048.9000
0.0550	9.19824	0.13131	1691.6500
0.0575	9.06569	0.134368	2287.0100
0.0600	8.94058	0.137492	2840.2800
0.0625	8.82222	0.140691	3355.9700
0.0650	8.71001	0.143973	3837.9600

Table 6. Sensitivity study of unit holding cost $\boldsymbol{h}$

h	v	t_1	$T.A.C.$
0.54	9.7748	0.1069	-881.1870
0.57	9.6267	0.1162	-237.6390
0.60	9.4890	0.1253	352.5180
0.63	9.3605	0.1342	896.1080
0.66	9.2402	0.1428	1398.8100
0.69	9.1274	0.1512	1865.3900
0.72	9.0211	0.1593	2299.8900
0.75	8.9209	0.1673	2705.7400
0.78	8.8261	0.1751	3085.8900

Table 7. Sensitivity study of unit deterioration cost $\boldsymbol{d}$

d	v	t_1	$T.A.C.$
5.6	8.8631	0.1622	7211.1600
6.0	8.9742	0.1553	6132.9300
6.4	9.0824	0.1487	5028.2500
6.8	9.1878	0.1425	3897.5200
7.2	9.2906	0.1365	2741.0900
7.6	9.3909	0.1308	1559.3100
8.0	9.4890	0.1253	352.51800
8.4	9.5849	0.1201	-878.9870
8.8	9.6787	0.1150	-2134.9000

Table 8. Sensitivity study of unit shortage cost $\boldsymbol{s}$

s	v	t_1	$T.A.C.$
1.5	9.4853	0.1250	284.8070
1.6	9.4861	0.1251	298.3520
1.7	9.4868	0.1251	311.8960
1.8	9.4875	0.1252	325.4380
1.9	9.4882	0.1253	338.9790
2.0	9.4890	0.1253	352.5180
2.1	9.4897	0.1254	366.0560
2.2	9.4904	0.1254	379.5920
2.3	9.4912	0.1255	393.1270

Table 9. Sensitivity study of unit production cost $\boldsymbol{p}$

p	v	t_1	$T.A.C.$
1.6	9.4914	0.2422	346.1040
2.0	9.4908	0.2146	347.6630
2.4	9.4902	0.1913	348.9660
2.8	9.4898	0.1713	350.0680
3.2	9.4895	0.1540	351.0090
3.6	9.4892	0.1388	351.8180
4.0	9.4890	0.1253	352.5180
4.4	9.4888	0.1133	353.1260
4.8	9.4886	0.1026	353.6570

Table 10. Sensitivity study of unit lost sale cost $\boldsymbol{l}$

l	v	t_1	$T.A.C.$
2.40	9.3452	0.1152	-2300.3000
2.55	9.3816	0.1176	1632.1300
2.70	9.4177	0.1201	-967.2880
2.85	9.4535	0.1227	-305.7420
3.00	9.4890	0.1253	352.5180
3.15	9.5242	0.1280	1007.5100
3.30	9.5591	0.1308	1659.2400
3.45	9.5937	0.1337	2307.7300
3.60	9.6281	0.1366	2952.9900

Table 11. Sensitivity study of production coefficient $\boldsymbol{\alpha}$

α	v	t_1	$T.A.C.$
0.84	9.4956	0.3750	338.7030
0.90	9.4930	0.2977	343.0070
0.96	9.4915	0.2436	346.0530
1.02	9.4905	0.2032	348.3180
1.08	9.4898	0.1716	350.0570
1.14	9.4893	0.1462	351.4250
1.20	9.4890	0.1253	352.5180
1.26	9.4887	0.1077	353.4020
1.32	9.4885	0.0927	354.1240

Table 12. Sensitivity study of inflation rate $\boldsymbol{r}$

r	v	t_1	$T.A.C.$
0.035	9.5892	0.1202	-221.9760
0.037	9.5723	0.1210	-124.3730
0.040	9.5555	0.1219	-27.5211
0.042	9.5388	0.1227	68.5875
0.045	9.5221	0.1236	163.9590
0.047	9.5055	0.1244	258.6000
0.050	9.4890	0.1253	352.5180
0.052	9.4725	0.1262	445.7180
0.055	9.4560	0.1270	538.2060

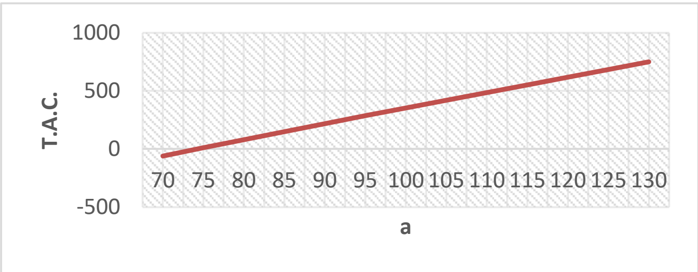


Figure 4. Graph of $\boldsymbol{T.A.C.}$ with respect to $\boldsymbol{a}$.

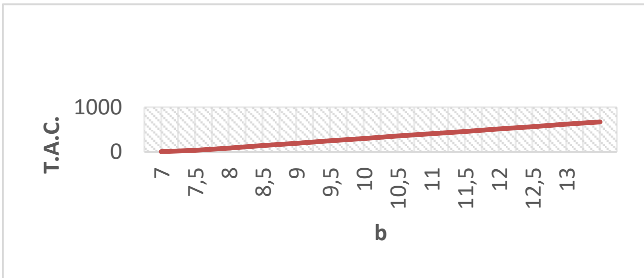


Figure 5. Graph of $\boldsymbol{T.A.C.}$ w. r. to $\boldsymbol{b}$.

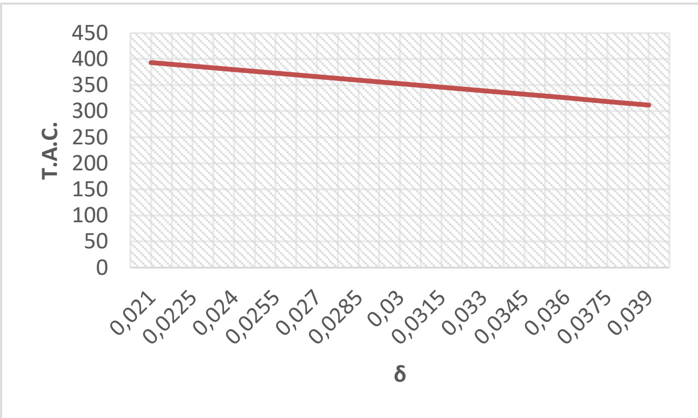


Figure 6. Graph of. $T.A.C.$ w. r. toδ.

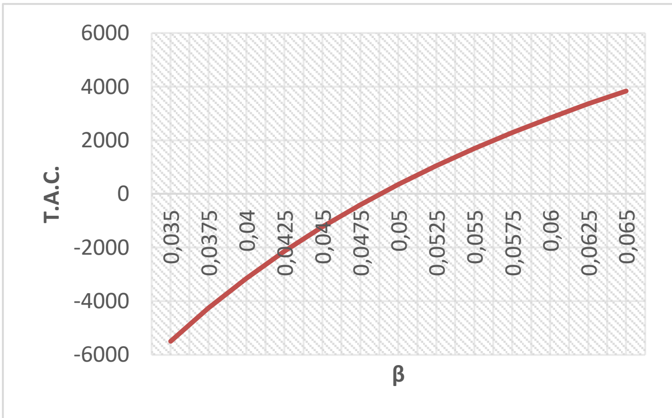


Figure 7. Graph of$T.A.C.$ w. r. toβ.

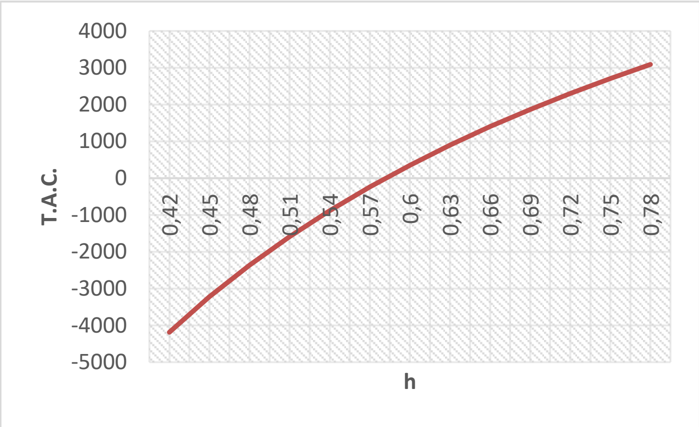


Figure 8. Graph of$T.A.C.$ w. r. to$h$.

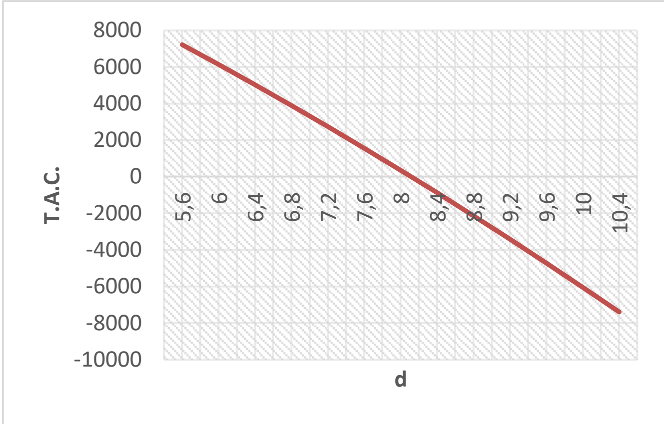
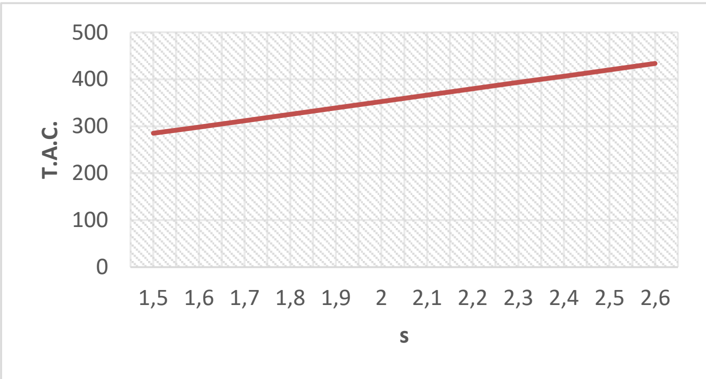


Figure 9. Graph of. $T.A.C.$ w. r. to$d$.



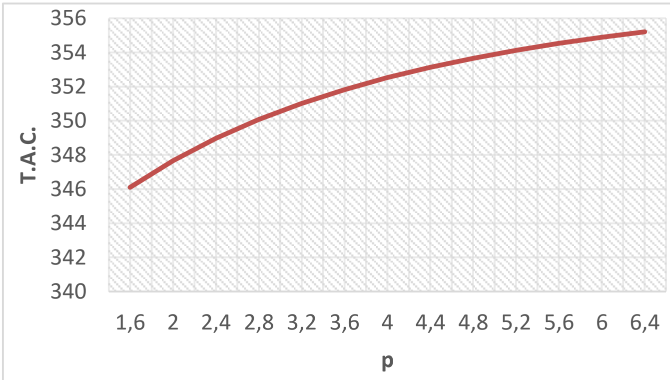


Figure 10. Graph of. $T.A.C.$ w. r. to$s$.

Figure 11. Graph of $T.A.C.$ w. r. to$p$.

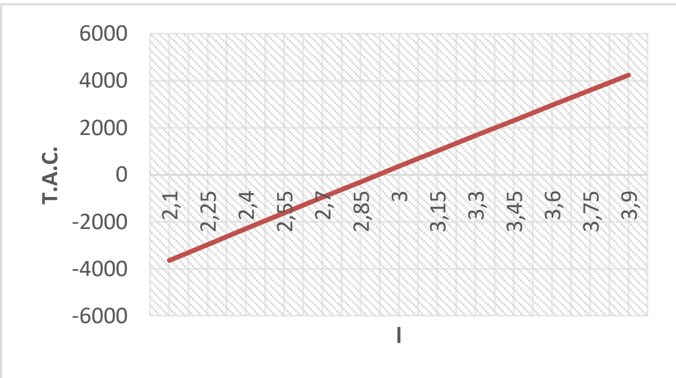


Figure 12. Graph of $T.A.C.$ w. r. to l.

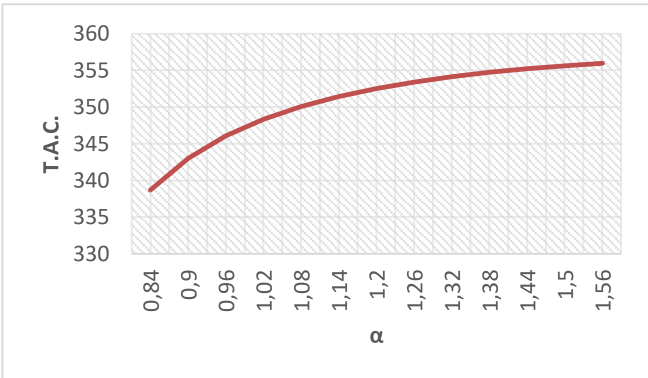


Figure 13. Graph of $T.A.C.$ w. r. toα.

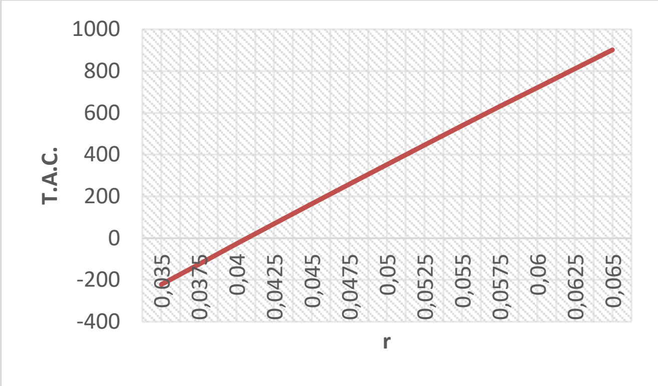


Figure 14. Graph of $T.A.C.$ w. r. to r.

10. Result and Discussion

With the help of the above sensitivity tables, the following observations were made.

- Table 2 and Table 3 demonstrates that on increasing the demand parameter "a" and "b" the value of the critical point "v" and the production period "t_1" decreases whereas the $T.A.C.$ increases.
- Table 4 shows that by increasing the backlogging rate "δ", and keeping the remaining parameters unchanged, we observe a decrement in the value of $T.A.C.$ Also we see that "v" and "t_1" are insensitive to the changes in "δ".
- Table 5 shows that with the rise in the value of the deterioration coefficient "β",$T.A.C.$ immediately starts decreasing.
- Table 6 shows that increasing the unit holding cost "h",$T.A.C.$ rises rapidly. Also, the decrease in $v$and increase in t_1 is observed.
- Table 7 shows that unit deterioration cost is directly proportional to "v". When the unit deterioration cost "v" decreases, "d" falls and $T.A.C.$ and "t_1" increases significantly.
- Table 8 shows the proportionality between unit shortage cost and $T.A.C.$ When the unit shortage cost "s" decreases, $T.A.C.$ also decreases and if the unit shortage cost "s" increases, $T.A.C.$ also increases and this does not affect the values of "v" and "t_1" .
- It is evident that if we increase the unit production cost, then $T.A.C.$ should increase; conversely, if we decrease the unit production cost, then $T.A.C.$ should decrease. This is shown in the sensitivity analysis table 9; when we change unit production cost "p" it is proportionately impacted $T.A.C.$ and does not affect the values of "v".
- Table 10 shows that unit lost sale cost "l" is directly proportional to "v", "t_1" and $T.A.C.$.
- Table 11 shows that change in the variation coefficient "α" directly impacted "v", and $T.A.C.$ i.e., when "α" increases, $T.A.C.$ also rises whereas "v" decreases.
- From the sensitivity analysis table 12, we see that the inflation rate "r" is directly proportional to $T.A.C.$ and this does not affect the values of "v" and "t_1".

11. Conclusions

This study obtains a realistic scenario in which a retailer must have sufficient inventory to meet customer demand in order to prevent inventory exhaustion and depletion as prices rise, because the retailer's reputation in the market depends on the stock of goods.

In this developed model, a two-parametric Weibull-type decay rate is taken. The demand rate was taken as exponential. We observed the role of inflation and considered the production rate a function of the demand rate. In this study, we determine an optimal policy that can deal with this situation. To illustrate this theory, we provide a mathematical framework of the model. Numerical examples are also given, and the convexity of the total average cost function is shown. The sensitivity analysis was used to validate the obtained model.

The limitations of this article may allow other researchers to continue our work by considering all upfront expenses and accounting for the business crisis, including the impact of inflation, by starting the review process before work begins. Furthermore, this study examines the proposed model in terms of price-based demand, inventory-based demand, etc. Models can also be developed for various delayed payment scenarios.

12. Acknowledgements

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13. Conflicts of Interest

The authors declare no conflict of interest.

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Appendix

The holding cost *(H.C.)* involved in this model is given as:

(a) $$H.C. = h\left\{\int_0^{t_1} e^{-rt}.I_1(t)dt + \int_{t_1}^{v} I_2(t)e^{-rt}dt\right\}$$

Putting the value of $I_1(t)$ and $I_2(t)$ in above equation, value of holding cost is given as

$$H.C. = h\left\{ a(\alpha-1)\begin{bmatrix}\left(\frac{1}{2}t_1^2 + \frac{b}{6}t_1^3 + \frac{\beta}{(\gamma+1)(\gamma+2)}t_1^{\gamma+2}\right) - r\left(\frac{1}{3}t_1^3 + \frac{b}{8}t_1^4 + \frac{\beta}{(\gamma+1)(\gamma+3)}t_1^{\gamma+3}\right) \\ -\beta\left(\frac{1}{(\gamma+2)}t_1^{\gamma+2} + \frac{b}{2(\gamma+3)}t_1^{\gamma+3} + \frac{\beta}{(\gamma+1)(2\gamma+2)}t_1^{2\gamma+2}\right)\end{bmatrix}\right.$$
$$Se^{\beta t_1^{\gamma}}\left((v-t_1) - \frac{r}{2}(v^2 - t_1^2) - \frac{\beta}{\gamma+1}(v^{\gamma+1} - t_1^{\gamma+1})\right) +$$

$$a\left(\begin{array}{l}\left(vt_1-\frac{v^2}{2}-\frac{t_1^2}{2}\right)+\frac{b}{2}\left(vt_1^2-\frac{v^3}{3}-\frac{2}{3}t_1^3\right)+\frac{\beta}{\gamma+1}\left(vt_1^{\gamma+1}-\frac{v^{\gamma+2}}{\gamma+2}-\frac{\gamma+1}{\gamma+2}t_1^{\gamma+2}\right)\\ -r\left[\left(\frac{t_1v^2}{2}-\frac{v^3}{3}-\frac{t_1^3}{6}\right)+\frac{b}{2}\left(\frac{t_1^2v^2}{2}-\frac{v^4}{4}-\frac{t_1^4}{4}\right)+\frac{\beta}{\gamma+1}\left(\frac{t_1^{\gamma+1}v^2}{2}-\frac{v^{\gamma+3}}{\gamma+3}-\frac{(\gamma+1)t_1^{\gamma+3}}{2(\gamma+3)}\right)\right]\\ -\beta\left[\begin{array}{l}\left(\frac{v^{\gamma+1}t_1}{\gamma+1}-\frac{v^{\gamma+2}}{\gamma+2}-\frac{t_1^{\gamma+2}}{(\gamma+1)(\gamma+2)}\right)+\frac{b}{2}\left(\frac{v^{\gamma+1}t_1^2}{\gamma+1}-\frac{v^{\gamma+3}}{\gamma+3}-\frac{2t_1^{\gamma+3}}{(\gamma+1)(\gamma+3)}\right)\\ +\frac{\beta}{\gamma+1}\left(\frac{v^{\gamma+1}t_1^{\gamma+1}}{\gamma+1}-\frac{v^{2\gamma+2}}{2\gamma+2}-\frac{(\gamma+1)t_1^{\gamma+3}}{(\gamma+1)(2\gamma+2)}\right)\end{array}\right]\end{array}\right)\Bigg\}.$$

(b) When $\mathbf{0 \le M \le t_1}$:

In this case, interest payable $= \boldsymbol{IP_1 = pi_p\left[\int_M^{t_1} I_1(t)e^{-rt}dt + \int_{t_1}^{v} I_2(t)e^{-rt}dt\right]}$

Putting the value of $I_1(t)$ and $I_2(t)$ in above equation, interest payable is given as

$$= pi_p\left\{a(\alpha-1)\left[\begin{array}{l}(t_1-M)+\frac{1}{3}\left(\frac{b}{2}-r\right)(t_1^3-M^3)-\frac{rb}{8}(t_1^4-M^4)-\frac{\beta\gamma}{(\gamma+1)(\gamma+2)}\left(t_1^{\gamma+2}-M^{\gamma+2}\right)\\ +\frac{2r\gamma\beta-\beta b(\gamma+1)}{2(\gamma+1)(\gamma+3)}\left(t_1^{\gamma+3}-M^{\gamma+3}\right)-\frac{\beta^2}{(\gamma+1)(2\gamma+2)}\left(t_1^{2\gamma+2}-M^{2\gamma+2}\right)\\ +\frac{r\beta b}{2(\gamma+4)}\left(t_1^{\gamma+4}-M^{\gamma+4}\right)+\frac{r\beta^2}{(\gamma+1)(2\gamma+3)}\left(t_1^{2\gamma+3}-M^{2\gamma+3}\right)\end{array}\right]\right.$$

$$+S\left[(v-t_1)-\frac{r}{2}(v^2-t_1^2)-\frac{\beta}{(\gamma+1)}\left(v^{\gamma+1}-t_1^{\gamma+1}\right)+\beta t_1^{\gamma}(v-t_1)-\frac{\beta r t_1^{\gamma}}{2}(v^2-t_1^2)+\frac{\beta r}{\gamma+2}\left(v^{\gamma+2}-t_1^{\gamma+2}\right)\right]$$

$$+a\left[\begin{array}{l}t_1(v-t_1)-\frac{1}{2}(v^2-t_1^2)-rt_1(v-t_1)+\frac{r}{3}(v^3-t_1^3)-\frac{\beta t_1}{(\gamma+1)}\left(v^{\gamma+1}-t_1^{\gamma+1}\right)+\frac{\beta}{(\gamma+2)}\left(v^{\gamma+2}-t_1^{\gamma+2}\right)+\\ \frac{\beta r t_1}{(\gamma+2)}\left(v^{\gamma+2}-t_1^{\gamma+2}\right)-\frac{\beta r}{(\gamma+3)}\left(v^{\gamma+3}-t_1^{\gamma+3}\right)\end{array}\right]$$

$$+\frac{b}{2}\left[\begin{array}{l}t_1^2(v-t_1)-\frac{1}{3}(v^3-t_1^3)-\frac{rt_1^2}{2}(v^2-t_1^2)+\frac{r}{4}(v^3-t_1^3)-\frac{\beta t_1^2}{(\gamma+1)}\left(v^{\gamma+1}-t_1^{\gamma+1}\right)+\frac{\beta}{(\gamma+3)}\left(v^{\gamma+3}-t_1^{\gamma+3}\right)+\\ \frac{\beta r t_1^2}{(\gamma+2)}\left(v^{\gamma+2}-t_1^{\gamma+2}\right)-\frac{\beta r}{(\gamma+4)}\left(v^{\gamma+4}-t_1^{\gamma+4}\right)\end{array}\right]$$

$$\left.+\frac{\beta}{(\gamma+1)}\left[\begin{array}{l}t_1^{\gamma+1}(v-t_1)-\frac{1}{(\gamma+2)}\left(v^{\gamma+2}-t_1^{\gamma+2}\right)-\frac{rt_1^{\gamma+1}}{2}(v^2-t_1^2)+\frac{r}{(\gamma+3)}\left(v^{\gamma+3}-t_1^{\gamma+3}\right)-\frac{\beta t_1^{\gamma+1}}{(\gamma+1)}\left(v^{\gamma+1}-t_1^{\gamma+1}\right)\\ +\frac{\beta}{(2\gamma+2)}\left(v^{2\gamma+2}-t_1^{2\gamma+2}\right)+\frac{\beta r t_1^{\gamma+1}}{(\gamma+2)}\left(v^{\gamma+2}-t_1^{\gamma+2}\right)-\frac{\beta r}{(2\gamma+3)}\left(v^{2\gamma+3}-t_1^{2\gamma+3}\right)\end{array}\right]\right\}$$

.

(c) When $\boldsymbol{t_1 \le M \le v}$:

In this case, interest payable $= IP_2 = pi_p\left[\int_M^v I_2(t)e^{-rt}dt\right]$

Putting value of $I_2(t)$ in the above equation

$$pi_p\left\{S\left[(v-M)-\frac{r}{2}(v^2-M^2)-\frac{\beta}{(\gamma+1)}\left(v^{\gamma+1}-M^{\gamma+1}\right)+\beta t_1^{\gamma}(v-M)-\frac{\beta r t_1^{\gamma}}{2}(v^2-M^2)+\frac{\beta r}{\gamma+2}\left(v^{\gamma+2}-M^{\gamma+2}\right)\right]+\right.$$

$$a\left[\begin{array}{l}t_1(v-M)-\frac{1}{2}(v^2-M^2)-rt_1(v-M)+\frac{r}{3}(v^3-M^3)-\frac{\beta t_1}{(\gamma+1)}\left(v^{\gamma+1}-M^{\gamma+1}\right)+\frac{\beta}{(\gamma+2)}\left(v^{\gamma+2}-M^{\gamma+2}\right)+\\ \frac{\beta r t_1}{(\gamma+2)}\left(v^{\gamma+2}-M^{\gamma+2}\right)-\frac{\beta r}{(\gamma+3)}\left(v^{\gamma+3}-M^{\gamma+3}\right)\end{array}\right]$$

$+$

$$\frac{b}{2}\left[\begin{array}{l} t_1^2(v-M)-\frac{1}{3}(v^3-M^3)-\frac{rt_1^2}{2}(v^2-M^2)+\frac{r}{4}(v^3-M^3)-\frac{\beta t_1^2}{(\gamma+1)}(v^{\gamma+1}-M^{\gamma+1})+\frac{\beta}{(\gamma+3)}(v^{\gamma+3}-M^{\gamma+3})+ \\ \frac{\beta r t_1^2}{(\gamma+2)}(v^{\gamma+2}-M^{\gamma+2})-\frac{\beta r}{(\gamma+4)}(v^{\gamma+4}-M^{\gamma+4}) \end{array}\right]$$

$$+$$

$$\left.\frac{\beta}{(\gamma+1)}\left[\begin{array}{l} t_1^{\gamma+1}(v-M)-\frac{1}{(\gamma+2)}(v^{\gamma+2}-M^{\gamma+2})-\frac{rt_1^{\gamma+1}}{2}(v^2-M^2)+\frac{r}{(\gamma+3)}(v^{\gamma+3}-M^{\gamma+3})-\frac{\beta t_1^{\gamma+1}}{(\gamma+1)}(v^{\gamma+1}-M^{\gamma+1}) \\ +\frac{\beta}{(2\gamma+2)}(v^{2\gamma+2}-M^{2\gamma+2})+\frac{\beta r t_1^{\gamma+1}}{(\gamma+2)}(v^{\gamma+2}-M^{\gamma+2})-\frac{\beta r}{(2\gamma+3)}(v^{2\gamma+3}-M^{2\gamma+3}) \end{array}\right]\right\}.$$