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ARTICLE

On Combining Ratio and Product Type Estimators For Estimation of Finite Population Mean In Adaptive Cluster Sampling Design

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Abstract

This article introduces a novel class of estimators and several new novel member estimators, combining the ratio and product forms, within the framework of Adaptive Cluster Sampling (ACS) design for estimating finite population mean. Specifically designed for rare or hidden clustered populations, the new novel estimators developed from the proposed class offer enhanced efficiency in estimation. To study the proposed class comprehensively, we derive expressions for the bias and Mean Squared Error (MSE) up to the first order of approximation. Through comprehensive simulation studies, we demonstrate the superior efficiency of the new developed estimators over several existing alternatives considered in this study.

Keywords: Sampling; Adaptive design; Adaptive cluster samplig; Ratio-cum-product; Class of estimators.

1. Introduction

In survey sampling, the sampling design to be used depends on the population that is under study. Mostly the sampling designs namely Simple random sampling, Stratified random sampling, Cluster sampling (among others) and their combinations are used in studies to obtain a representative sample. But, if the population under study is clumped or hidden clustered, non-adaptive sampling designs may not provide a representative sample. This issue was addressed by (Thompson, 1990). (Thompson, 1990) in 1990 proposed the Adaptive cluster sampling design that deals with rare or hidden clustered type populations. In the Adaptive cluster sampling (ACS) design, the units are selected subject to a condition or criteria which is determined by the researcher which helps in obtaining the hidden units.

Extensive use of auxiliary information in developing estimators started with work of (Cochran, 1940). Since then, various ratio, product and regression type estimators have been proposed. Making an exhaustive list of all such estimators might be fruitless. However, here we cite some papers which in our view introduced new concepts in the domain of developing estimators and are noteworthy (Bahl & Tuteja, 1991; Grover & Kaur, 2011; Gupta & Shabbir, 2008; Khoshnevisan *et al.,* 2007; Singh & Solanki, 2013; Singh *et al.,* 2016). In ACS design, the work of developing efficient estimators picked up the pace post the work of (Dryver & Chao, 2007) where they proposed the transformed population approach. (Dryver & Chao, 2007) put forth the idea that ACS can be considered as SRSWOR from the transformed population. Since then, various estimators have been proposed. (Dryver & Chao, 2007) first proposed the ratio estimator in the ACS design. (Chutiman, 2013) proposed some ratio type estimators based on single auxiliary variable using some parameters of auxiliary variable. (Yadav *et al.,* 2016) proposed some improved ratio estimators of population mean in ACS using single auxiliary variable and known coefficients of skewness, kurtosis and correlation. (Qureshi *et al.,* 2018) developed a generalized estimator using single auxiliary variable with some known parameters of auxiliary variable and some known robust parameters.

(Singh *et al.,* 2016) proposed a ratio-cum-product type estimator which allows the researchers a freedom to not rigidly opt for a ratio or a product type estimator based on information on correlation between survey and auxiliary variables. Such estimators should be developed more and therefore in this paper, we have developed a generalized class of ratio-cum-product type estimators in the ACS design by combining the ratio and product form and studied its properties.

The article is organized as follows. In Section 2, we provide the proposed generalized ratio-cum-product type class along with its derivations of bias and MSE (Mean square error). Further, we present some new estimators developed from the proposed generalized ratio-cum-product type class. In Section 3, a simulation study has been carried out to demonstrate the performance of the developed estimators compared with the competing estimators presented in the Appendix. In Section 4 we provide the results and concluding remarks on the study along with future areas of research.

2. Proposed generalized ratio-cum-product type class

Motivated from (Singh *et al.,* 2016; Dryver & Chao, 2007) we propose the following class:

$$t_G = \bar{w}_y \left(\frac{a\mu x + b}{a\bar{w}_x + b}\right)^{2g} \left(\frac{a\mu x + b}{a\bar{w}_x + b}\right)^{-1} = \bar{w}_y \left(\frac{a\mu x + b}{a\bar{w}_x + b}\right)^{(2g-1)}, \tag{1}$$

where a and b are suitably choosen to produce different estimators (existing and new estimators) for instance, Mid range, Hodges-Lehman, coefficient of skewness, coefficient of kurtosis and coefficient of variation while g is optimised such that the MSE of t_G is minimum.
It is noteworthy to note that many prominent estimators that are considered in this article (see Table-4) are members of the proposed class t_G as follows:

1. t_{TH} (Thompson, 1990) $\in t_G$ for $g = \frac{1}{2}$.
2. t_{DC} (Dryver & Chao, 2007) $\in t_G$ for $a = 1$, $b = 0$ and $g = 1$.
3. t_{CH_1} (Chutiman, 2013) $\in t_G$ for $a = 1$, $b = C_{w_x}$ and $g = 1$.
4. t_{CH_2} (Chutiman, 2013) $\in t_G$ for $a = \beta_2(w_x)$, $b = C_{w_x}$ and $g = 1$.
5. t_{CH_3} (Chutiman, 2013) $\in t_G$ for $a = 1$, $b = \beta_2(w_x)$ and $g = 1$.

6. t_{YS_1} (Yadav *et al.*, 2016) $\in t_G$ for $a = \beta_2(w_x)$, $b = \beta_1(w_x)$ and $g = 1$.
7. t_{YS_2} (Yadav *et al.*, 2016) $\in t_G$ for $a = \beta_1(w_x)$, $b = \beta_2(w_x)$ and $g = 1$.
8. t_{QK_1} (Qureshi *et al.*, 2018) $\in t_G$ for $a = MR$, $b = \beta_1(w_x)$ and $g = 1$.
9. t_{QK_2} (Qureshi *et al.*, 2018) $\in t_G$ for $a = MR$, $b = TM$ and $g = 1$.
10. t_{QK_3} (Qureshi *et al.*, 2018) $\in t_G$ for $a = HL$, $b = \beta_1(w_x)$ and $g = 1$.
11. t_{QK_4} (Qureshi *et al.*, 2018) $\in t_G$ for $a = HL$, $b = TM$ and $g = 1$.

Consider the following notations:
$e_{w_y} = \frac{\bar{w_y}}{\mu_y}$-1, $e_{w_x} = \frac{\bar{w_x}}{\mu_x} - 1$ such that $E(e_{w_y}) = E(e_{w_x}) = 0$,
$E(e_{w_y}^2) = fC_{w_y}^2$, $E(e_{w_x}) = fC_{w_x}^2$,
$E(e_{w_y w_x}) = f\rho_{w_y w_x} C_{w_y} C_{w_x}$ where $C_{w_y}^2 = \frac{S_{w_y}^2}{\mu_y^2}$, $C_{w_x}^2 = \frac{S_{w_x}^2}{\mu_x^2}$,
$\rho_{w_y w_x} = \frac{S_{w_x w_y}}{S_{w_y} S_{w_x}}$, $S_{w_x w_y} = \frac{1}{N-1}\sum_{i=1}^{N}(w_{y_i} - \mu_y)(w_{x_i} - \mu_x)$
$S_{w_y}^2 = \frac{1}{N-1}\sum_{i=1}^{N}(w_{y_i} - \mu_y)^2$ and
$S_{w_x}^2 = \frac{1}{N-1}\sum_{i=1}^{N}(w_{x_i} - \mu_x)^2$.
Using the above standard error term of ACS we re-write (1) as:

$$t_G = \mu_y(e_{w_y} + 1)(1 + \tau e_{w_x})^{(1-2g)}, \tag{2}$$

where $\tau = \frac{a\mu_x}{a\mu_x + b}$.
Further simplifying, we get:

$$t_G - \mu_y = \mu_y\left(-(2g-1)\tau e_{w_x} + g(2g-1)\tau^2 e_{w_x}^2 + e_{w_y} - (2g-1)\tau e_{w_y} e_{w_x}\right). \tag{3}$$

Taking exprectation on both sides, we obtain the bias as:

$$Bias(t_G) = \mu_y f\tau(2g-1)\left(g\tau - \rho_{w_y w_x}\frac{C_{w_y}}{C_{w_x}}\right)C_{w_x}^2. \tag{4}$$

Squaring and taking expectation on both sides of (3), we get:

$$MSE(t_G) = \mu_y^2 f\left(C_{w_y}^2 + (2g-1)\tau\left((2g-1)\tau - 2\rho_{w_y w_x}\frac{C_{w_y}}{C_{w_x}}\right)C_{w_x}^2\right). \tag{5}$$

To obtain the optimum value of g such that the MSE of proposed generalized class t_G is minimum, we partially differentiate (5) with respect to g and equate the resultant mathematical equation to zero and obtain:

$$g_{opt} = \frac{1}{2}\left[1 + \frac{\rho_{w_y w_x} C_{w_y}}{\tau C_{w_x}}\right]. \tag{6}$$

Using (6) in (5) we get the minimum MSE of the proposed generalized class t_G as

$$MSE(t_{G_{min}}) = fS_{w_y}^2(1 - \rho_{w_y w_x}^2). \tag{7}$$

Therefore we can say that using the optimum value of g the MSE of the proposed generalized ratio-cum-product type class t_G reduces to MSE of regression estimator (Chutiman, 2013). Note that, the optimum value of g requires known values of C_{w_y}, C_{w_x} and $\rho_{w_y w_x}$ which if not known can be obtained incurring nominal cost. From the proposed generalized ratio-cum-product class, we developed new estimators using some robust measure and known parameters of auxiliary variables. These new developed estimators have been presented in Table-1.

Table 1. New estimators generated from proposed class t_G

$t_{G_{new}}$	Form	a	b	τ
t_{G_1}	$\bar{w}_y\left(\frac{MR\mu x+\beta_1(w_x)}{MR\bar{w}_x+\beta_1(w_x)}\right)^{(2g_1-1)}$	MR	$\beta_1(w_x)$	$\frac{MR\mu_x}{MR\mu_x+\beta_1(w_x)}$
t_{G_2}	$\bar{w}_y\left(\frac{HL\mu x+\beta_1(w_x)}{HL\bar{w}_x+\beta_1(w_x)}\right)^{(2g_2-1)}$	HL	$\beta_1(w_x)$	$\frac{HL\mu_x}{HL\mu_x+\beta_1(w_x)}$
t_{G_3}	$\bar{w}_y\left(\frac{HL\mu x+\beta_2(w_x)}{HL\bar{w}_x+\beta_2(w_x)}\right)^{(2g_3-1)}$	HL	$\beta_2(w_x)$	$\frac{HL\mu_x}{HL\mu_x+\beta_2(w_x)}$
t_{G_4}	$\bar{w}_y\left(\frac{MR\mu x+\beta_2(w_x)}{HL\bar{w}_x+\beta_2(w_x)}\right)^{(2g_4-1)}$	MR	$\beta_2(w_x)$	$\frac{MR\mu_x}{MR\mu_x+\beta_2(w_x)}$
t_{G_5}	$\bar{w}_y\left(\frac{MR\mu x+C_{w_x}}{HL\bar{w}_x+C_{w_x}}\right)^{(2g_5-1)}$	MR	C_{w_x}	$\frac{MR\mu_x}{MR\mu_x+C_{w_x}}$

3. Simulation Study

In this section, we demonstrate the performance of our new developed estimators $t_{G_1}-t_{G_5}$ which we developed from the proposed generalized ratio-cum-product type class t_G. We compare the performance of $t_{G_1}-t_{G_5}$ with the competing existing estimators which are presented in Table-4 of the Appendix. For performance evaluation the criteria of PRE is used. To conduct the simulation study, following algorithm in R is used:

1. For generation of population, the model $y_i = \frac{x_i}{4} + e_i$ is used where $e_i \sim N(0, x_i)$ and observations on auxiliary variable X are taken from (Thompson, 2012).
2. Using the ACS procedure, samples of sizes $n = 130, 135, 140, 144$ are obtained and several values of all estimators are obtained.
3. For each sample size the MSE values are obtained using the formula $MSE = \frac{1}{20000}\sum_{i=1}^{20000}(t_{i*} - \mu_y)^2$ where t_* is the relevant estimator.
4. Using the MSE values obtained, the PRE values are calculated and presented in Table-[2, 3].

Table 2. PREs of all estimators in case of Population-1

	$n = 130$	$n = 135$	$n = 140$	$n = 144$
t_{TH}	100.0000	100.0000	100.0000	100.0000
t_{DC}	225.4049	228.5502	230.4671	232.6150
t_{CH_1}	131.8930	131.0003	132.2640	133.0798
t_{CH_2}	220.7595	219.6225	220.8657	221.7883
t_{CH_3}	114.6095	114.1245	115.1842	116.0060
t_{YS_1}	223.3870	222.3528	223.5499	224.4786
t_{YS_2}	140.1314	139.0677	140.4155	141.2335
t_{QK_1}	211.9429	210.5582	211.9308	212.8367
t_{QK_2}	225.4049	228.5502	230.4671	232.6150
t_{QK_3}	227.7656	226.9456	228.0575	228.9994
t_{QK_4}	225.4049	228.5502	230.4671	232.6150
t_{G_1}	240.8250	241.9288	242.4476	243.3371
t_{G_2}	240.3797	241.6450	242.2837	243.2588
t_{G_3}	240.9512	241.8853	242.2865	243.1052
t_{G_4}	240.7318	241.5559	241.8803	242.6663
t_{G_5}	240.8714	241.9488	242.4489	243.3260

Table 3. PREs of all estimators in case of Population-2

	n = 130	n = 135	n = 140	n = 144
t_{TH}	100.0000	100.0000	100.0000	100.0000
t_{DC}	283.9305	284.8197	285.5378	289.3675
t_{CH_1}	133.1603	133.0035	133.2082	132.8396
t_{CH_2}	241.0806	241.6793	241.6413	243.0329
t_{CH_3}	114.9675	114.8708	115.1820	114.7587
t_{YS_1}	245.1887	245.8487	245.8044	247.3266
t_{YS_2}	142.0040	141.8397	142.0039	141.6833
t_{QK_1}	228.1371	228.5561	228.5370	229.5564
t_{QK_2}	283.9305	284.8197	285.5378	289.3675
t_{QK_3}	252.4078	253.1808	253.1247	254.8933
t_{QK_4}	283.9305	284.8197	285.5378	289.3675
t_{G_1}	289.2581	290.1906	290.6799	294.8402
t_{G_2}	289.2875	290.2921	290.6861	294.8076
t_{G_3}	288.5690	289.4106	290.0666	294.2603
t_{G_4}	287.5362	288.3126	289.1319	293.3439
t_{G_5}	289.2025	290.1218	290.6320	294.7980

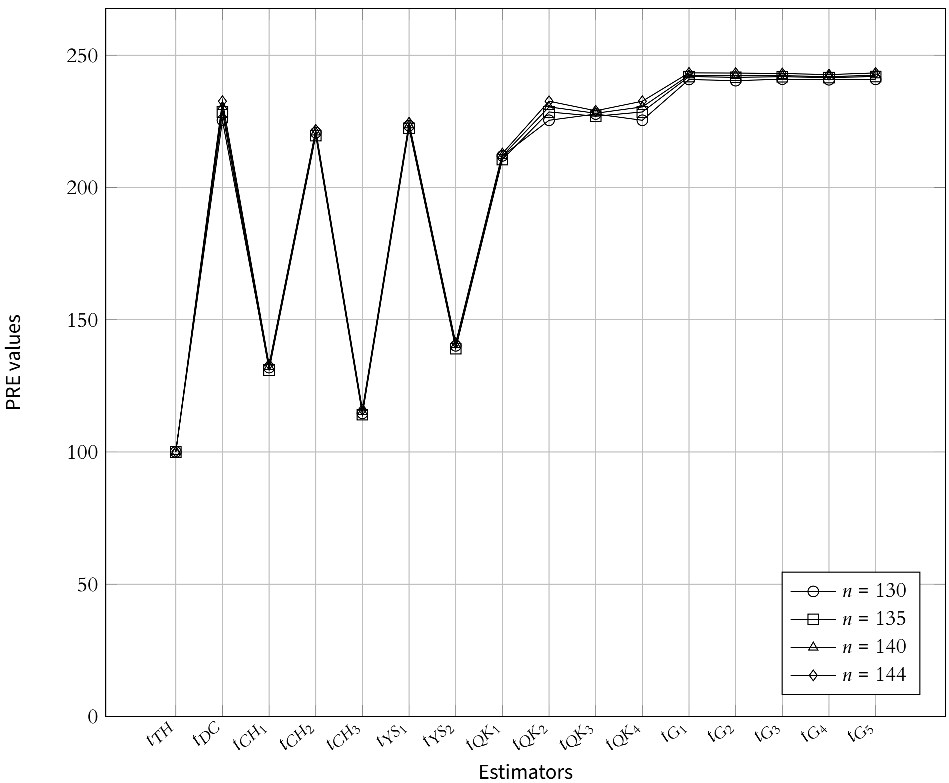

Figure 1. Line chart comparing PRE values of all estimators for Population-1.

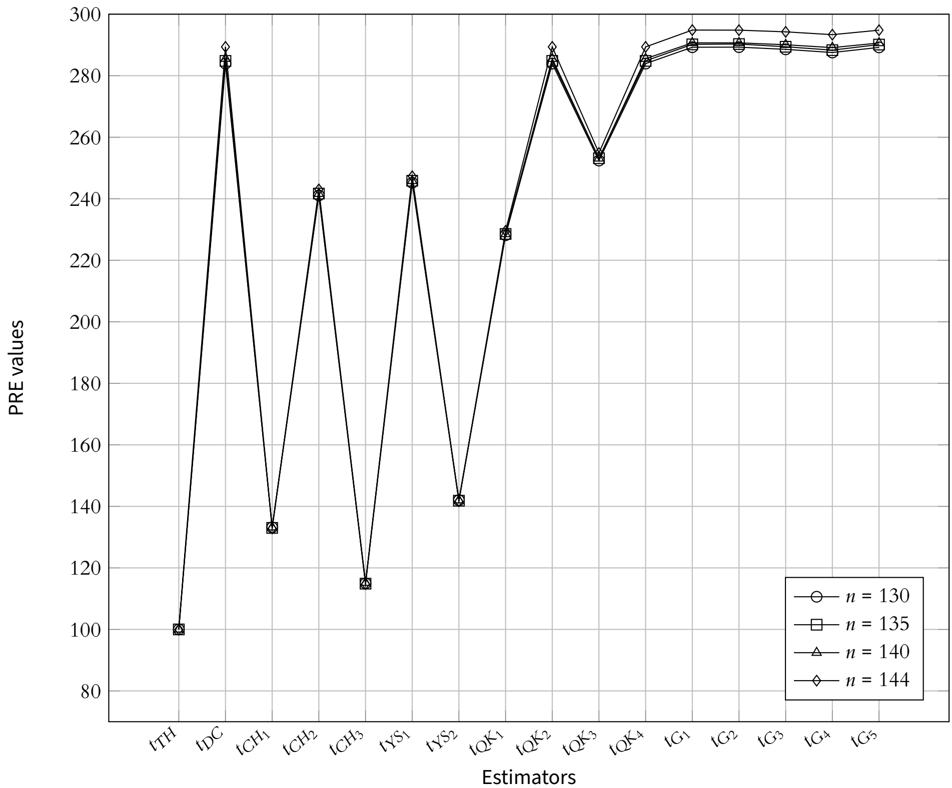

Figure 2. Line chart comparing PRE values of all estimators for Population-2.

4. Results and Discussion

The estimator that should be used depends on some factors for instance, available informations on paramters of the auxiliary variable, number of auxiliary variables available, correlation coefficient between the survey variable and the auxiliary variable(s). Our focus in this article was to deal with cases where an estimator can be used irrespective of what the correlation between survey and auxiliary variable is. Therefore Motivated by (Singh *et al.,* 2016) we proposed a generalized class of ratio-cum-product type estimators by combining the ratio and product form of estimators. In order to study this class, we derived the expressions of bias and MSE and presented their formulae. Further, we developed some new estimators $t_{G_{1-5}}$ from the proposed generalized class t_G. The new developed estimators $t_{G_{1-5}}$ utilize known population parameters of the auxiliary variable X namely Mid range, Hodges Lehman, coefficient of skewness, coefficient of kurtosis and the coefficient of variation. It is noteworthy that many prominent estimators which are considered in this article namely t_{Th}(Thompson, 1990), t_{DC}(Dryver & Chao, 2007), $t_{CH_1-CH_3}$(Chutiman, 2013), $t_{YS_1-YS_2}$(Yadav *et al.,* 2016) and $t_{QK_1-QK_4}$(Qureshi *et al.,* 2018) are members of the proposed generalized ratio-cum-product type class t_G.

To compare the performance of the new dveloped estimators $t_{G_{1-5}}$ with the similar competing estimators presented in this article (see Table-4), two simulation studies have been conducted. From the results obtained, we can see that all the new developed estimators $t_{G_{1-5}}$ performed better than the existing similar competing estimators presented in this article (see Figures [1, 2]). It can bee seen that the PREs of all the new developed estimators $t_{G_{1-5}}$ are well above the PREs of the similar existing competing estimators (see Tables -[2, 3]). It should be noted that among the new developed estimators $t_{G_{1-5}}$, the performance of each new developed estimator is almost the same (see Table-2,3). The minimum MSE of the proposed class t_G is equal to the minimum MSE of regression estimator in ACS based on single auxiliary variable (Chutiman, 2013) and this minimum MSE requires information on τ, $\rho_{w_y w_x}$, C_{w_y} and C_{w_x}. Therefore we recommend the use of our new estimators from class t_G when the population under study is rare and hidden clustered and ACS design is to be applied. Some future areas of research should be to develop estimators involving multiple auxiliary variable.

Conflicts of Interest

The authors declare no conflict of interest.

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5. Appendix

Here in this table we present the competing estimators and their Mean Squared Error (MSE.

Table 4. Competing estimators and their Mean Squared Error (MSE)

Estimator	MSE
$t_{Th} = \bar{w}_\gamma$ (Thompson, 1990)	$fS^2_{w_\gamma}$
$t_{DC} = \frac{\bar{w}_\gamma}{\bar{w}_x}\mu_x$ (Dryver & Chao, 2007)	$f\mu_Y^2(C^2_{w_\gamma} + C^2_{w_x} - 2\rho_{w_x w_\gamma} C_{w_\gamma} C_{w_x})$
$t_{CH_1} = \bar{w}_\gamma\left(\frac{\mu_x+C_{w_x}}{\bar{w}_x+C_{w_x}}\right)$ (Chutiman, 2013)	$f\mu_Y^2(C^2_{w_\gamma} + \left(\frac{\mu_x}{\mu_x+C_{w_x}}\right)^2 C^2_{w_x} - 2\left(\frac{\mu_x}{\mu_x+C_{w_x}}\right)\rho_{w_x w_\gamma} C_{w_\gamma} C_{w_x})$
$t_{CH_2} = \bar{w}_\gamma\left(\frac{\beta_2(w_x)\mu_x+C_{w_x}}{\beta_2(w_x)\bar{w}_x+C_{w_x}}\right)$ (Chutiman, 2013)	$f\mu_Y^2(C^2_{w_\gamma} + \left(\frac{\mu_x\beta_2(w_x)}{\mu_x\beta_2(w_x)+C_{w_x}}\right)^2 C^2_{w_x} - 2\left(\frac{\mu_x\beta_2(w_x)}{\mu_x\beta_2(w_x)+C_{w_x}}\right)\rho_{w_x w_\gamma} C_{w_\gamma} C_{w_x})$
$t_{CH_3} = \bar{w}_\gamma\left(\frac{\mu_x+\beta_2(w_x)}{\bar{w}_x+\beta_2(w_x)}\right)$ (Chutiman, 2013)	$f\mu_Y^2(C^2_{w_\gamma} + \left(\frac{\mu_x}{\mu_x+\beta_2(w_x)}\right)^2 C^2_{w_x} - 2\left(\frac{\mu_x}{\mu_x+\beta_2(w_x)}\right)\rho_{w_x w_\gamma} C_{w_\gamma} C_{w_x})$
$t_{YS_1} = \bar{w}_\gamma\left(\frac{\beta_2(w_x)\mu_x+\beta_1(w_x)}{\beta_2(w_x)\bar{w}_x+\beta_1(w_x)}\right)$ (Yadav *et al.*, 2016)	$f\mu_Y^2(C^2_{w_\gamma} + \left(\frac{\beta_2(w_x)\mu_x}{\beta_2(w_x)\mu_x+\beta_1(w_x)}\right)^2 C^2_{w_x} - 2\left(\frac{\beta_2(w_x)\mu_x}{\beta_2(w_x)\mu_x+\beta_1(w_x)}\right)\rho_{w_x w_\gamma} C_{w_\gamma} C_{w_x})$
$t_{YS_2} = \bar{w}_\gamma\left(\frac{\beta_1(w_x)\mu_x+\beta_2(w_x)}{\beta_1(w_x)\bar{w}_x+\beta_2(w_x)}\right)$ (Yadav *et al.*, 2016)	$f\mu_Y^2(C^2_{w_\gamma} + \left(\frac{\beta_1(w_x)\mu_x}{\beta_1(w_x)\mu_x+\beta_2(w_x)}\right)^2 C^2_{w_x} - 2\left(\frac{\beta_1(w_x)\mu_x}{\beta_1(w_x)\mu_x+\beta_2(w_x)}\right)\rho_{w_x w_\gamma} C_{w_\gamma} C_{w_x})$
$t_{QK_1} = \bar{w}_\gamma\left(\frac{MR\mu_x+\beta_1(w_x)}{MR\bar{w}_x+\beta_1(w_x)}\right)$ (Qureshi *et al.*, 2018)	$f\mu_Y^2(C^2_{w_\gamma} + \left(\frac{MR\mu_x}{MR\mu_x+\beta_1(w_x)}\right)^2 C^2_{w_x} - 2\left(\frac{MR\mu_x}{MR\mu_x+\beta_1(w_x)}\right)\rho_{w_x w_\gamma} C_{w_\gamma} C_{w_x})$
$t_{QK_2} = \bar{w}_\gamma\left(\frac{MR\mu_x+TM}{MR\bar{w}_x+TM}\right)$ (Qureshi *et al.*, 2018)	$f\mu_Y^2(C^2_{w_\gamma} + \left(\frac{MR\mu_x}{MR\mu_x+TM}\right)^2 C^2_{w_x} - 2\left(\frac{MR\mu_x}{MR\mu_x+TM}\right)\rho_{w_x w_\gamma} C_{w_\gamma} C_{w_x})$
$t_{QK_3} = \bar{w}_\gamma\left(\frac{HL\mu_x+\beta_1(w_x)}{HL\bar{w}_x+\beta_1(w_x)}\right)$ (Qureshi *et al.*, 2018)	$f\mu_Y^2(C^2_{w_\gamma} + \left(\frac{HL\mu_x}{HL\mu_x+\beta_1(w_x)}\right)^2 C^2_{w_x} - 2\left(\frac{HL\mu_x}{HL\mu_x+\beta_1(w_x)}\right)\rho_{w_x w_\gamma} C_{w_\gamma} C_{w_x})$
$t_{QK_4} = \bar{w}_\gamma\left(\frac{HL\mu_x+TM}{HL\bar{w}_x+TM}\right)$ (Qureshi *et al.*, 2018)	$f\mu_Y^2(C^2_{w_\gamma} + \left(\frac{HL\mu_x}{HL\mu_x+TM}\right)^2 C^2_{w_x} - 2\left(\frac{HL\mu_x}{HL\mu_x+TM}\right)\rho_{w_x w_\gamma} C_{w_\gamma} C_{w_x})$